Toward Smooth Analysis of Robotic Contact Tasks Problem

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Abstract

In robotic tasks where the manipulator has to make transition from free space motion to constrained one, there always exists an inevitable phase transition. A number of controllers have been proposed in the literature with various discussions on their practical implications. In this paper first a discussion on the nature of the structure of a practical contact transition controller is presented. Then a novel framework which can be used toward studying and analysis of such controllers for robotic contact tasks problem is proposed. This framework presents a natural set-up to study the performance of the controller. The framework is based on the Flippov's notion of differential inclusion and a definition of the smooth Lyapunov function.

1. Introduction

Various initiatives have been focused on the development of control laws which can result in an stable controller for the manipulator during the transition from free motion to constrained motion. Kazerooni [1] proposed a controller based on the notion of impedance control where the closed-loop dynamics of the manipulator is *matched* with a target dynamics with compliance properties. The stability of the controller is then investigated based using the small gain theorem. Mills [2] proposed a discontinuous controller and discussions on its performance based on the notion of the generalized dynamical systems. In this scheme, the environment is modeled as a spring and damper (a compliant model of the environment [6]). A practical approach for controlling the contact transition was proposed by Payandeh [3][12]. In this method, the gains of the controller are switched during different phases of motion to achieve a stable contact force regulation. Several methods for controlling the transition from free motion to constrained motion with the objective on minimizing fingertip load oscillation during transition was investigated by Hyde and Cutkosky[4]. A suitable controller based on the input command shaping was then proposed. Using one degree of freedom manipulator, experiments were performed to demonstrate the stable performance of the controller. Marth, Tarn and Bejczy [5] proposed a model-based algorithm combined with explicit force control to regulating the phase transition. The force controller is switched on when the contact is established. The stability results were presented for the impact phase of the manipulator with the rigid environment. The non-collocation effects between sensors and actuators have been studied in [7], [8]. A thorough study of force robotic force controllers has been provided in [9], [10]. A more complete review of various impact models and controllers are also given in[11].

As correctly discussed by Kazerooni, Mills and Payandeh, in contact transition, the dynamics of the closed-loop system changes and the external contact force measurements which was omitted as a part of feedback control system in the free motion is now a part of the controller. However, the methodology of using the force signal as a natural indication of the contact phenomenon has been incorporated in different ways. Mills has use the force signal to switch the controller from the position control PD mode to the force control PD mode. While Kazerooni used the force signals as a part of the feedback loop while maintaining the structure of the controller unchanged (i.e. impedance control). In this way, the position controller can also be used as a force control through the impedance parameterization. Payandeh has used the measure of the

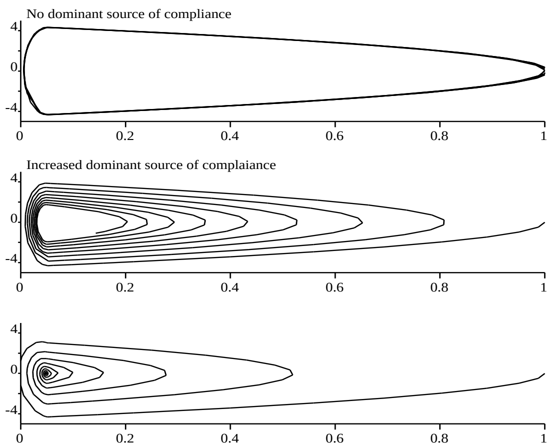

Figure 1: Simulation results of an ideal 1DOF manipulator when interacting with a rigid environment with various degrees of compliance.

instant of contact to change the gains of the controller while maintaining the structure of the robust controller fixed during the phases of free motion, impact and constrained motion.

The paper is organized as follows: section (2) presents an overview of modelling of physical systems which results in an implementation of a bounce-less contact transition controller; in section (3) presents smooth stability analysis of the non-smooth system using 1DOF example and finally section (5) presents concluding remarks.

2. Implementation of a bounce-less Controller

In general in has been realized that to accomplish a practical bounce-less controller in robotic contact tasks, there has to exist some compliance either in the structure of the manipulator (e.g. structural flexibility), in the contacting environment (e.g. compliant environment) or in the closed-loop controller (e.g. stiffness controller). This presence of compliance either in the contacting environment or in the manipulator structure can reduce the effective impulsive forces which arises due to the collision between two solid bodies[14]. The advantages of the presence of compliance has been shown both analytically and experimentally, see for example, [1], [2], [6],[13], [9], [10] and [11]. This in effect reduces the contact dynamic from the infinite mode to a finite mode and can reduce (eliminate) the number of bounces. For example, Figure (1) shows a simulated response of an ideal 1DOF manipulator when approaching a stiff environment with the presence of various degree of compliance.

In general, compliance in the contacting bodies (either in the environment or manipulator) allows the increase in the closed-loop bandwidth and hence increasing the bounds of the controller gains [8]. For example, Figure (2) and (3) show the actual response of a 2DOF manipulator when approaching a rigid environment with an identical controllers. It can be seen for the case where there is no dominant compliance presence, the manipulator bounces from the contacting environment (positive value of approximately 1 lbf represents the non-contact phase, any value less than this represents the contact phase). Detailed structure of this controller can be found in [3].

In the following, 1DOF stability analysis of this type of controller is proposed based on the Flippov's notion of the differential inclusions and smooth Lyapunov function candidate.

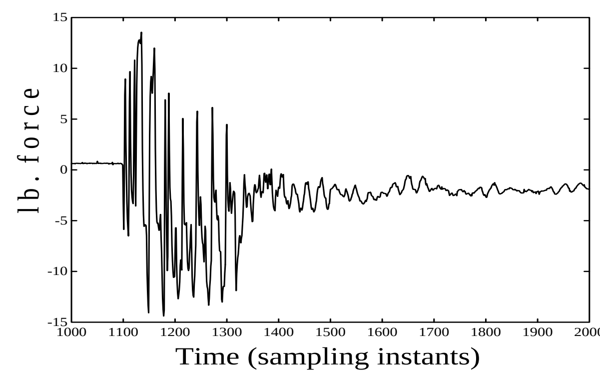

Figure 2: Contact force response of the manipulator during the impact phase in the case where no dominant compliance is introduced(sampling frequency of 2500HZ).

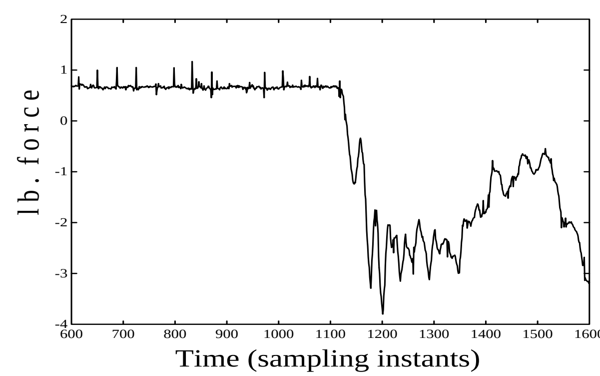

Figure 3: Contact force response of the manipulator during the impact phase having the same approach velocity of that of figure (2) with the introduction of dominant source of compliance.

3. Toward Smooth Analysis of a Non-Smooth System

Let us consider a one dimensional compliant mechanism with the actuation force. The dynamic equation

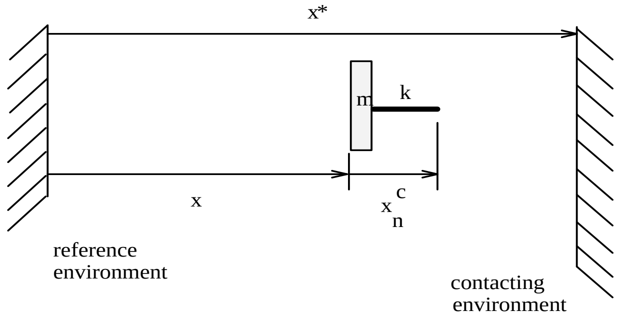

Figure 4: Definition of the variables associated with the one-dimensional example

of the system in an unconstrained phase can be written as:

$$m\ddot{x}(t) = -r + u(t)^1 \tag{1}$$

where for this case the reaction force from the environment during the contact phase is zero, or $r = 0$. The dynamic equation of this system in contact with a rigid environment can be written as:

$$m\ddot{x}(t) = -r + u(t)^2 \tag{2}$$

where $r = k_c(x_n^c - x^\star + x)$ for $x^\star - x < x_n^c$ is the reaction force acting on the mass due to interaction with the rigid environment and the k_c is a model of compliant. Figure (4) shows all the position variables involved.

Let us now define a controller for both contact and non-contact phases of the mechanism of the form:

$$u(t) = \begin{cases} u(t)^1 = -k_p(x - x^d) - k_d\dot{x} \ , \ r < 0 \\ u(t)^2 = -k_{pf}(r - r^d) - k_{df}\dot{r} + r^d \ , \ r > 0 \end{cases} \tag{3}$$

where $x^d, \dot{x}^d$ is the desired trajectory of the end-point and r^d is the desired contact force to be exerted on the environment (i.e. x^d is generated through an exploratory planning strategy to located the unknown environment). At the instant when the contact is detected, the controller regulates the desire contact force specified by r^d. By taking into account the relationship between the contact force and the state variables of the mechanisms, the controller $u(t)^2$ can be written as:

$$u(t)^2 = k_{pf}k_c(x - x^d) - k_{df}k_c\dot{x} + r^d \tag{4}$$

In general, the dynamic equations describing the model of the object in both regions are discontinuous as the object contacts the environment. The discontinuity arises from the presence of the contact force from the environment as well as the nature of the controller. Such a dynamical system is called a non-smooth dynamical system. The conventional solution theories to differential equations are no longer valid for these type of systems. Filippov proposed a solution concept for non-smooth systems(referred to here as Filippov's solution). The properties of these solutions were studied systematically which included a comparison between Filippov's solution and conventional solution and also theorems on the existence and uniqueness of the solutions. In this work, Filippov's solution concept is used and existence and uniqueness of the solution proven followed by stability analysis.

3.1. Existence and Uniqueness of Filippov's Solution

The two dynamic models of the system in both regions can be written in term of the state space model (assuming $x_1 = x$):

$$\begin{array}{rcl} (a)\dot{x}_1 & = & x_2 \\ (b)\dot{x}_2 & = & -\frac{r}{m} + \frac{u(t)}{m} \end{array} \tag{5}$$

The right-hand side of the equation (5b) is discontinuous. The discontinuity surface can be represented as follows:

$$S := \{(x_1, x_2), x_1 - x^\star + x_n^c = 0\} \tag{6}$$

Equation (6) represents the contact surface between the manipulator and environment.

We apply Flippov's solution theory to prove the existence and uniqueness of the solution to the proposed system shown in equation (5b). The right-hand side of the equation (5b) is piecewise continuous. The conditions for existence and continuity of the Flippov's solution, such as the right-hand sides of equations(5) are measurable and bounded, are both satisfied. Thus, the existence and continuity of the solutions to equations (5) are guaranteed.

We are to prove the uniqueness of the solution to our system. The solution region Ω is divided by the discontinuity surface S into two parts, $\Omega^+ := \{(x_1, x_2) : x - x^\star + x_n^c > 0\}$ where the manipulator is in contact with the environment, and $\Omega^- := \{(x_1, x_2) : x - x^\star + x_n^c < 0\}$ where it is in free motion. The right-hand side of equation (5) in both regions are f^+ and f^- which are defined as: $f^+ =$.

$$\begin{Bmatrix} x_2 \\ (-k_c(x_1 - x^d) - k_{pf}k_c(x_1 - x^d) - k_{df}k_c x_2)/m \end{Bmatrix}$$
$$f^- = \begin{Bmatrix} x_2 \\ (-k_p(x_1 - x^d) - k_d x_2)/m \end{Bmatrix} \tag{7}$$

According to Lemma 9 of Flippov[15], the projection of f^+ and f^- along the normal to the discontinuity surfaces, which is defined as N_s, $N_s = \{1, 0\}^T$, need to be examined. Such projections are denoted by f_N^+ and f_N^-, respectively. For example under study, we have that $f_N^+ = f_N^- = x_2$. Since f_N^+ and f_N^- have the same sign, the uniqueness of Flippov's solution for equation (5) is guaranteed. Furthermore, when $x_2 > 0$, the solution trajectory goes from Ω^- to Ω^+ and has only one point in common with S. When $x_2 < 0$, the solution trajectory goes from Ω^+ to Ω^- and has only one point in common with S. In summary, the existence and uniqueness of Flippov's solution for the non-smooth control system described by equation (5) are proved.

3.2. Stability Analysis

Lyapunov's stability theory was developed for smooth dynamic systems. The extensions of Lyapunov's stability theory to non-smooth dynamic systems, has been previously studied by Solncev[16], Hahn[17], and Shevitz and Paden[18]. Their extensions were based on the belief that non-smooth Lyapunov function arise naturally for non-smooth systems. Another recent extension was developed by Wu et. al[19] , in that conditions for the construction of smooth Lyapunov functions for classes of non-smooth dynamic system were established.

Assuming $e_1 = x_1 - x^d$, the state-space model can be written as:

$$\begin{array}{rcl} \dot{e}_1 & = & e_2 \\ \dot{e}_2 & = & \frac{k_p}{m}e_1 + \frac{k_d}{m}e_2 \end{array} \tag{8}$$

and

$$\begin{array}{rcl} \dot{e}_1 & = & e_2 \\ \dot{e}_2 & = & -\frac{k_{pf}}{m}(k_c+1)e_1 - \frac{k_{df}}{m}k_c e_2 \end{array} \tag{9}$$

Equations (8) and (9) describe the manipulator in both non-contact region and contact region, respectively. The discontinuity surface is defined as:

$$S := \{(e_1, e_2); e_1 + x^d + x_n^c - x^\star = 0\} \tag{10}$$

When $e \in \Omega^+$, the manipulator is in contact with the environment and when $e \in \Omega^-$, the manipulator is not in contact with the environment. A Lyapunov function is now constructed in each region as:

$$V = \left\{ \begin{array}{l} V_1 = \frac{1}{2}e_2^2 + \frac{k_p}{2m}e_1^2 + \frac{(k_{pf}+1)k_c - k_p}{2m}(x^\star - x_n^c - x^d)^2 \\ \qquad \text{for } (e \in \Omega^-) \\ V_2 = \frac{1}{2}e_2^2 + \frac{k_c+1}{2m}k_{pf}e_1^2 \qquad (e \in \Omega^+) \\ V_3 = \frac{1}{2}e_2^2 + \frac{k_c+1}{2m}k_{pf}e_1^2 \qquad (e \in S) \end{array} \right. \tag{11}$$

Function V is positive through the entire region under the condition that $(k_{pf}+1)k_c - k_p > 0$. Function V is definite since $V = 0($ i.e. $V_2 = 0)$ if and only if $e_1 = 0$ and $e_2 = 0$. Function V is continuous on the discontinuity surface. This can be shown as follow: $lim_{x \to S^-} V_1 =$

$$lim_{e_1 \to (x^d + x_n^c - x^\star)^-} V = \frac{1}{2}e_2^2 + \frac{k_{pf}+1}{2m}k_c(x^\star - x_n^c - x^d)^2$$

$lim_{x \to S^+} V_2 =$

$$lim_{e_1 \to (x^d + x_n^c - x^\star)^+} V_2 = \frac{1}{2}e_2^2 + \frac{k_p+1}{2m}k_c e_1^2$$

or: $lim_{x \to S^+} V_2 = \frac{1}{2}e_2^2 + \frac{k_p+1}{2m}k_c(x^\star - x_n^c - x^d)^2$. From the above two equations, we can see that as the solution trajectory approaches the discontinuity surface, $lim_{e \to S^-} V_1 = lim_{e \to S^+} V_2 = V_3$ (note that $e_1 = x^\star - x_n^c - x^d$ on the discontinuity surface). Therefore, the Lyapunov function in equation (11) is continuous on the discontinuity surface. The derivative of the above Lyapunov function with respect to time is:

$$\dot{V} = \left\{ \begin{array}{l} \dot{V}_1 = -k_d e_2^2 \text{ for, } (e \in \Omega^-) \\ \dot{V}_2 = -k_{df} k_c e_2^2 \text{ for, } (e \in \Omega^+) \end{array} \right.$$

$\dot{V}$ is negative in both non-contact and contact regions. On the discontinuity surface where $e_1 = x^\star - x_n^c - x^d$, we have $\partial V(e \in S)$:

$$= co\left\{ \left[\begin{array}{c} \frac{k_p}{m}(x^\star - x_n^c - x^d) \\ e_2 \end{array} \right], \left[\begin{array}{c} \frac{k_c+1}{m}k_{pf}(x^\star - x_n^c - x^d) \\ e_2 \end{array} \right] \right\}.$$

Accordingly, Flippov's differential inclusion is: $K[f](e \in S)$:

$$co\left\{ \left[\begin{array}{c} e_2 \\ -\frac{k_p}{m}(x^\star - x_n^c - x^d) - \frac{k_d}{m}e_2 \end{array} \right], \left[\begin{array}{c} e_2 \\ -\frac{k_c+1}{m}k_{pf}(x^\star - x_n^c - x^d) - \frac{k_{df}}{m}k_c e_2 \end{array} \right] \right\}.$$

Let $\{\xi, e_2\}$ be an arbitrary element of $\partial V(e \in S)$, then:

$$\dot{V}(e \in S) \in \bigcap_{\{\xi, e_2\} \in \partial V} \{\xi, e_2\} K[f](e \in S)$$

Let us denote that $K[\dot{V}] = \{\xi, e_2\}K[f](e \in S)$. We have to prove that all the elements of $\bigcap_{\{\xi, e_2\} \in \partial V} K[\dot{V}]$ are either negative or zero. Therefore, $\dot{V}(e \in S)$ is either negative or zero.

The set $K[\dot{V}]$ can be rewritten as follows:

$$K[\dot{V}] = co\{e_2(\xi - \frac{k_p}{m}(x^\star - x_n^c - x^d)) - \frac{k_d}{m}e_2^2, \\ e_2(\xi - \frac{k_c - 1}{m}k_{pf}(x^\star - x_n^c - x^d)) - \frac{k_{df}}{m}k_c e_2^2\}$$

or,

$$K[\dot{V}] = [e_2(\xi - \frac{k_p}{m}(x^\star - x_n^c - x^d)) - \frac{k_d}{m}e_2^2, \\ e_2(\xi - \frac{k_c + 1}{m}k_{pf}(x^\star - x_n^c - x^d)) - \frac{k_{df}}{m}k_c e_2^2]$$

and

$$\dot{V}(e \in S) \in \bigcap_{\{\xi, e_2\} \in \partial V} K[\dot{V}]$$

Here we only prove one special case that $e_2 > 0$. For $e_2 < 0$, similar proof can be applied. Since $(k_{pf} + 1)k_c - k_p > 0$, we have:

$$\frac{k_p}{m}(x^\star - x_n^c - x^d) \leq \xi \leq \frac{k_c + 1}{m} k_{pf}(x^\star - x_n^c - x^d)$$

Thus, the following relations hold:

$$\xi - \frac{k_p}{m}(x^\star - x_n^c - x^d) \geq 0$$
$$\xi - \frac{k_c+1}{m} k_{pf}(x^\star - x_n^c - x^d) \leq 0$$

If $k_d = k_{df} k_c, \bigcap_{\{\xi, e_2\} \in \partial V} K[\dot{V}](\xi) = \{-\frac{k_d}{m} e_2^2\}$, that is set $\bigcap_{\{\xi, e_2\} \in \partial V} K[\dot{V}](\xi)$ contains only one element as $-\frac{k_d}{m} e_2^2$. Thus, $\dot{V}(e \in S) = -\frac{k_d}{m} e_2^2$, which is negative. From the above discussion, we can see that $\dot{V}$ is either negative or zero through the whole region. Thus the control system is stable.

4. Conclusions and Future Work

This paper present some basic experimental results in implementing a contact task controller. The controller takes advantage of a source of compliance which is present in the manipulating system. The overall controller consists of a collection of the controllers where they can be switched as a function of force signal measurements, time or state preconditions. A novel stability results of this contact task controller for the case of 1DOF system are presented. The framework is based on the notion of the Flippov's differential inclusion and the definition and the construction of the smooth Lyapunov function. The future work involves extension of this 1DOF analysis to the case of non-smooth multi-degree of freedom dynamical systems.

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