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ON PROPERTIES OF THE SCREW GEOMETRY OF CONTACT WRENCHES IN MULTIPLE GRASPING AGENTS

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ABSTRACT

Establishing a force-closure contact configuration between multiple contacting agents such as dexterous fingers and an object is one of the main requirements for further object manipulation. Various tools and methods have been proposed in the literature for such analysis and modelling. This paper extends the previous results by the authors and by using basic notions from *screw geometry* present some preliminary tools for a) locating the next contact wrench given any first two contact wrenches; b) computing the grasping wrenches as a function of the external wrench (force) and c) computing the local friction wrenches between the agents and object as a function of the external wrench. The main contribution of the paper is the application of basic tools from screw geometry and the associated algebra to the problem of planning and analysis of forces in multiple contacting agents.

1 INTRODUCTION

There has been a considerable body of literature in the area of positioning the contact points on the object such that when it is grasped, there exists an equilibrium between all the forces[1][2][3]. In general, the condition of force equilibrium (or force-closure) reduces to determining the rank condition of a matrix, i.e. grasp matrix . If the matrix has a full rank, then the configuration of the grasp allows any external forces to be counter-acted by the grasping forces. The upper magnitude of grasping forces (i.e. internal forces) are usually determined based on optimization methods[1][2][4]. In general, the upper magnitudes of grasping force are selected which can ensure robustness of the grasp such that the object does not have any local slip between the agents. This approach is usually very conservative since in general the fingers at all time have to exert these forces on the object. Although the above approaches have been followed by many researches for further enhancements and extensions, there has been very few who considered the geometrical interpretation of the forces in the multi-agent contact in determining various force-closure properties and the grasp configuration[5][6][7][8][9]. This paper extends the previous definitions in the context which may give some geometrical insight in determining a configuration of contacts between the agents which can result in a force-closure condition. In addition, it is shown how any external force can be decomposed into the components of contact forces and further into the net friction forces which are required in order to avoid the slippage of the object between the agents. The proposed method results in a closed form formulation where it is shown an analytical form exists for such mappings. The results of this paper can be incorporated at various stages in grasp planning and object manipulation algorithms, e.g.[14][15][16]. The paper addresses the following problems in grasp configuration in multiple agents:

- Given two independent grasping forces on the object, what is a condition which the third grasping force must satisfy in order for it to be linearly independent than the other two.
- Given the grasping force vectors on the object which are linearly independent, what is the closed-form solution which can expand any external force into its compo-

nents along the grasping forces.

- What are the relationships between the external forces and the friction forces at the contact points.

The paper is organized as follow: section (2) presents some definitions regarding the notion of screw geometry and its associated algebra; section (3) presents a method for determining the location of third contacting wrench on the object for obtaining the force-closure properties; section (4) presents a method for determining the magnitude of grasping wrenches in a force-closure grasp as a function of the external wrench; section (5) summarizes an approach for determining the direction and magnitude of the friction forces between the three agents and object and finally section (6) presents concluding remarks.

2 PRELIMINARIES

In this section we present a review of some basic definitions from mechanics with their generalization in terms of screw geometry[10][11]. Let us define the following vector description between any given two points (p, q) in a rigid body as:

$$\mathbf{X}(q) = \mathbf{X}(p) + \omega_{\mathbf{X}} \times \vec{pq} \tag{1}$$

where $\mathbf{X}(.)$ and $\omega_{\mathbf{X}}$ are vectors $\in \mathbf{R}^3$. The above relationship is a generalization of what has been referred to as the moment field $\mathbf{M}$ in dynamics and velocity field (or infinitesimal displacement) $\mathbf{V}$ in kinematic description of rigid bodies. For example, one can state that the resultant force and moment acting on a rigid body can be represented as a *resultant force* $\omega_{\mathbf{X}} = \mathbf{f}$ along a line(axis) of action and the *resultant moment* $\mathbf{X}(p) = \mathbf{m}$ about the axis. Similarly, the resultant motion of a rigid body can be represented by a linear velocity $\mathbf{v}$ along an axis and the angular velocity ω about the axis. These definitions from mechanics of rigid bodies are referred to as a *wrench* $\mathbf{F}$ and *twist* $\mathbf{T}$ of a body respectively about a *screw axis* in screw geometry. Or,

$$\mathbf{F} = (\mathbf{f}; \mathbf{m}) \qquad \mathbf{T} = (\omega; \mathbf{v}) \tag{2}$$

where $\mathbf{f}$ is a force vector and $\mathbf{m}$ is the moment vector; ω is the angular velocity vector and $\mathbf{v}$ is the linear velocity vector. The associated moment field $\mathbf{M}$ and the velocity field $\mathbf{V}$ in terms of equation (1) can be expressed as:

$$\mathbf{M}(p) = \mathbf{m} + \mathbf{f} \times \vec{op} \qquad \mathbf{V}(p) = \mathbf{v} + \omega \times \vec{op}$$

where p is any point in the moment or velocity field in the rigid body and o is any point on the $\omega_{\mathbf{X}}$ axis.

The vector field defined in equation (1) is generally referred to as a *skew-symmetric* or *helicoidal* vector field $\mathcal{D}$[9][10][11]. To this vector field there is associated algebraic operations. For example, let us define a vector $\mathbf{U}$ in terms of two vectors $\mathbf{X}$ and $\mathbf{Y}$ belonging to $\mathcal{D}$ as:

$$\mathbf{U} = \omega_{\mathbf{X}} \times \mathbf{Y}(p) - \omega_{\mathbf{Y}} \times \mathbf{X}(p) \tag{3}$$

where (p) is any point. The above operation is analogous to ordinary vector cross product. This operation is written as:

$$\mathbf{U} = [\mathbf{X}, \mathbf{Y}] \tag{4}$$

which is referred to as the Lie bracket operation e.g. see [10][11]. Another algebraic operation which is defined in $\mathcal{D}$ is the definition of the inner product (or Klein form) between two of its members (this operation is analogous to the ordinary scalar product):

$$[\mathbf{X} \mid \mathbf{Y}] = \omega_{\mathbf{X}} \cdot \mathbf{Y}(p) + \omega_{\mathbf{Y}} \cdot \mathbf{X}(p) \tag{5}$$

Two members of $\mathcal{D}$ are said to be reciprocal to each other if $[\mathbf{X} \mid \mathbf{Y}] = 0$. This property has been extensively used in the formulation of this paper. Another operation is referred to as the *Killing form* is defined as:

$$\Omega \mathbf{X}(p) = \omega_{\mathbf{X}} \tag{6}$$

for example for the moment field the above operation will result in the force vector associated with the field and for the velocity field it is the angular velocity vector. In dual vector formulation (see for example [11][12]), the operator Ω is the multiplication by ϵ. For example, let us define a member of $\mathcal{D}$ in form of dual vector or: $\mathbf{F} = \mathbf{f} + \epsilon \mathbf{m}$ and let us for now assume that the pitch of such skew-symmetric field is zero. Hence, for a unit dual vector, $\mathbf{F}$ can be written in terms of its Plucker line coordinates of its screw axis, or $\mathbf{f} = (L, M, N)$ and $\mathbf{m} = (P, Q, R)$. From equation (6) we have $\epsilon \mathbf{F} = \epsilon \mathbf{f}$ where we have $\epsilon^2 = 0$ and $\omega_{\mathbf{X}} = f$.

For the formulation of this paper, we also define the following operation:

$$(\mathbf{X} \mid \mathbf{Y}) = [\mathbf{X} \mid \Omega \mathbf{Y}] \equiv [\Omega \mathbf{X} \mid \mathbf{Y}]$$

Analogous to dual vector, the dual number z (i.e. member of dual number ring Δ) [11][12] is defined as $\hat{z} = x + \epsilon y$

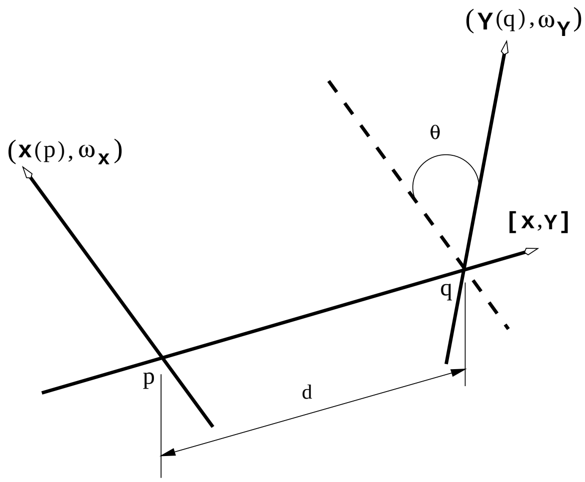

Figure 1. Definition of $[\mathbf{X}, \mathbf{Y}]$.

with x and y are real numbers and with the property that $\epsilon^2 = 0$. The real and dual part of z are denoted by $Re(\hat{z}) = x$ and $Du(\hat{z}) = y$. Hence, the product of $\mathbf{X} \in \mathcal{D}$ by a scalar $z \in \Delta$ is defined as:

$$\hat{z}\mathbf{X} = x\mathbf{X} + y\Omega\mathbf{X}$$

we also define *dual inner product* of two members of $\mathcal{D}$ as a dual coefficient combination of the Killing and Klein inner product, or:

$$\{\mathbf{X} \mid \mathbf{Y}\} = (\mathbf{X} \mid \mathbf{Y}) + \epsilon[\mathbf{X} \mid \mathbf{Y}]$$

with the definition of the Lie bracket, we define the *dual triple product* by:

$$\{\mathbf{X}; \mathbf{Y}; \mathbf{Z}\} = \{\mathbf{X} \mid [\mathbf{Y}, \mathbf{Z}]\} \tag{7}$$

As an example, let us define two members of $\mathcal{D}$ in a form of dual vector representation as: $\mathbf{F}_1$ and $\mathbf{F}_2$. Then the Lie bracket between these two member in terms of dual vector formulation can be written as:

$$\mathbf{F}_3 = [\mathbf{F}_1, \mathbf{F}_2] = \mathbf{f}_1 \times \mathbf{f}_2 + \epsilon(\mathbf{f}_1 \times \mathbf{m}_2 + \mathbf{m}_1 \times \mathbf{f}_2) \tag{8}$$

$[\mathbf{F}_1, \mathbf{F}_2]$ can be interpreted as a wrench that acts on a screw whose axis is the common perpendicular to the axes of both $\mathbf{F}_1$ and $\mathbf{F}_2$, see Figure (1).

3 LOCATING A THIRD CONTACT POINT ON THE OBJECT

In general, contact locations between three agents and an object is determined such that beside the grasp configuration satisfying the force-closure requirement, it may also has to satisfy other task requirements[2]. This section uses the screw geometry of any given two grasping wrenches and defines a condition for determining the location and orientation of the third grasping wrench on the object.

Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be two linearly independent wrenches representing the two grasping force vectors. The question is where to position and orient the third grasping force on the object in such a way that it is linearly independent than the other two? In general it is stated that three linearly independent grasping forces will always result in a force closure grasp (i.e. in a three point contact with friction, the rank of the grasp matrix is equal to 6).

There are many approaches which can be followed to locate the position and orientation of the third contact force. Here, we utilize the geometrical properties which exist between wrench representation of the contact forces. Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be in $\mathcal{D}$, then we define $\mathbf{F}_3 = [\mathbf{F}_1, \mathbf{F}_2]$ (equation (4)). This states that $\mathbf{F}_3$ can be obtained as a Lie bracket of the other given two contacting wrenches.

By definition, the three wrench representation of the forces $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are linearly independent. In summary we can state that:

- $\mathbf{F}_1$ and $\mathbf{F}_2$ are linearly independent over Δ[1]
- $\epsilon\mathbf{F}_1$ and $\epsilon\mathbf{F}_2$ are linearly independent over real numbers $\mathbf{R}$;
- $\{\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3\}$ are the basis of $\mathcal{D}$ over Δ.

The implication of the above condition is that the orientation of $\mathbf{F}_3$ can be determined as a line parallel to $\mathbf{U} = [\mathbf{F}_1, \mathbf{F}_2]$. Essentially the above statement presents a method where by finding the intersection of $\mathbf{U}$ with the surface presentation of the object, one can determine a possible location of the third contacting wrench $\mathbf{F}_3$.

For example, let $\mathbf{F}_3$ be determined as in equation (8). In general, the screw axis representing the wrench $\mathbf{F}_3$ can be defined in terms of Plucker line coordinates as: $\mathbf{F}_3 = (\mathbf{f}; \mathbf{m}) = (L, M, N; P^\star, Q^\star, R^\star)$. A vector locating a point on the axis can be determined as: $l = (\mathbf{f} \times \mathbf{m}) \in \mathbf{R}^3$. Give a vector l expressed with respect to x, y and z coordinate frame, the projected parametric equations of the screw axis can then be written as: $x = l_x + Lt$, $y = l_y + Mt$ and $z =$

[1] The set of vectors $\{r_1, r_2, \cdots, r_n\}$; is linearly independent over $\mathbf{R}$ if:

$$\lambda_1 r_1 + \lambda_2 r_2 + \cdots + \lambda_n r_n = 0$$

where $\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$ (λ_i is a real number). The set of vectors $\{r_1, r_2, \cdots, r_n\}$ are linearly independent over Δ is the same property is verified for dual numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$.

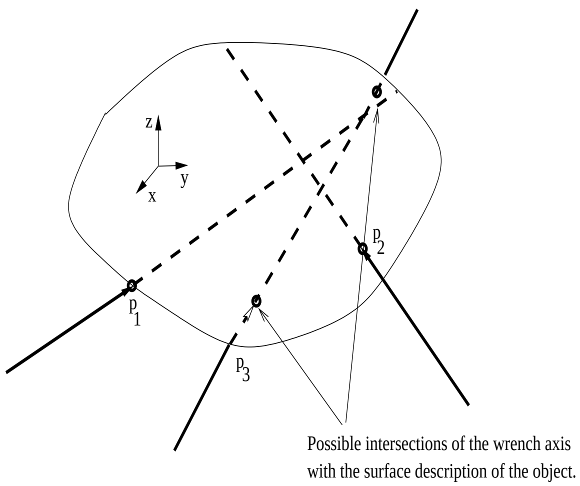

Figure 2. Definition of $[\mathbf{X}, \mathbf{Y}]$.

$l_z + Mt$ where (l_x, l_y, l_z) are the projection of vector l and t is a free parameter. It is now easy to find the intersection of this parametric line with surface representation of an object. Depending on the surface representation, there may be more than one solution associated with the location of the third grasping point (Figure (2)). In general, another layer of planning has to be incorporated for selecting the feasible contact points. A simple example of locating such intersection point can be found in [13].

4 MAPPING OF AN EXTERNAL WRENCH TO LINEARLY INDEPENDENT GRASPING WRENCHES

Let the three grasping wrenches $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ be in $\mathcal{D}$. If these wrenches are linearly independent, then we have:

i) $\{\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3\}$ is a basis of $\mathcal{D}$ over Δ.
ii) $\{\epsilon\mathbf{F}_1, \epsilon\mathbf{F}_2, \epsilon\mathbf{F}_3\}$ is linearly independent over space of real number $\mathbf{R}$.
iii) any external wrench $\mathbf{W} \in \mathcal{D}$, can be expressed as $\mathbf{W} = \hat{f}_1\mathbf{F}_1 + \hat{f}_2\mathbf{F}_2 + \hat{f}_3\mathbf{F}_3$

where $\hat{f}_i$ (member of Δ) is a dual number multiplier of the grasping wrench $\mathbf{F}_i$. $\hat{f}_1$,$\hat{f}_2$ and $\hat{f}_3$ can be determined as:

$$\begin{aligned} \hat{f}_1 &= \tfrac{1}{G}\{\mathbf{W}; \mathbf{F}_2; \mathbf{F}_3\} \\ \hat{f}_2 &= \tfrac{1}{G}\{\mathbf{W}; \mathbf{F}_3; \mathbf{F}_1\} \\ \hat{f}_3 &= \tfrac{1}{G}\{\mathbf{W}; \mathbf{F}_1; \mathbf{F}_2\} \end{aligned} \tag{9}$$

where $G = \{\mathbf{F}_1; \mathbf{F}_2; \mathbf{F}_3\}$, $\hat{f}_1 = f_1 + \epsilon f_{1o}$, $\hat{f}_2 = f_2 + \epsilon f_{2o}$ and $\hat{f}_3 = f_3 + \epsilon f_{3o}$ where the expansion of $\{\cdot;\cdot;\cdot\}$ is given in equation 7.

For example, let the expansion of the external wrench be given as:

$$\mathbf{W} = \hat{f}_1\mathbf{F}_1 + \hat{f}_2\mathbf{F}_2 + \hat{f}_3\mathbf{F}_3 \tag{10}$$

To determine $\hat{f}_1$ we multiply both sides of equation (10) by $[\mathbf{F}_2, \mathbf{F}_3]$, we have:

$$\{\mathbf{W} \mid [\mathbf{F}_2, \mathbf{F}_3]\} = \hat{f}^1\{\mathbf{F}_1 \mid [\mathbf{F}_2, \mathbf{F}_3]\} \tag{11}$$

Let $\mathbf{P} = [\mathbf{F}_2, \mathbf{F}_3]$. By definition, $\mathbf{P} \in \mathcal{D}$. The axis of $\mathbf{P}$ is perpendicular to the axis of $\mathbf{F}_2$ and $\mathbf{F}_3$. Hence, it can be shown $[\mathbf{F}_2 \mid \mathbf{P}] = 0$ and $[\mathbf{F}_3 \mid \mathbf{P}] = 0$. Hence, $\mathbf{P}$ is reciprocal to both $\mathbf{F}_2$ and $\mathbf{F}_3$. Then from equation (10) we can solve for the dual number $\hat{f}^1$ as:

$$\hat{f}_1 = \frac{\{\mathbf{W} \mid [\mathbf{F}_2, \mathbf{F}_3]\}}{\{\mathbf{F}_1 \mid [\mathbf{F}_2, \mathbf{F}_3]\}}$$

Similarly, one can solve for $\hat{f}^2$ and $\hat{f}^3$. Note that $\{\mathbf{F}_1 \mid [\mathbf{F}_2, \mathbf{F}_3]\} = \{\mathbf{F}_2 \mid [\mathbf{F}_3, \mathbf{F}_1]\} = \{\mathbf{F}_3 \mid [\mathbf{F}_1, \mathbf{F}_2]\} = G$ because of symmetry of the inner product.

5 DETERMINATION OF FRICTION FORCES

Here the problem can be stated as given the magnitude of any external forces acting on the object, what are the frictional components of the grasping forces which are required in order to result in force equilibrium. Hence, as a part of the iterative planning algorithm, the magnitude of the grasping forces (i.e. normal component of the force) can be adjusted such that as a function of the local friction coefficients, the slippage of the object can be avoided. As a result, depending on the various manipulation tasks, the grasping forces can be adaptively modified to counter balance the external forces. Aspects of this problem are addressed in [5][6] and the following is an extension to some of the previous results.

Here we assume that the local contact model follows the Coulomb friction model and the grasping force $\mathbf{F}_i \in \mathcal{D}$ can be projected into normal and tangential components using object local geometrical information, i.e. surface normals. The question is that for a given external wrench and the components of normal forces, what would be the required local friction forces. The problem can be formulated as:

$$\mathbf{W} - \sum_{i=1}^{3} n_i\mathbf{N}_i = \mathbf{W}_T = \sum_{i=1}^{3} f_i\mathbf{F}_i \tag{12}$$

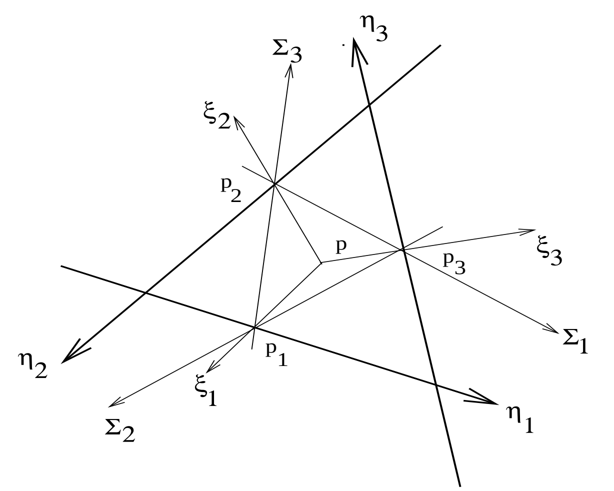

Figure 3. Definition of screw representation of the grasp geometry.

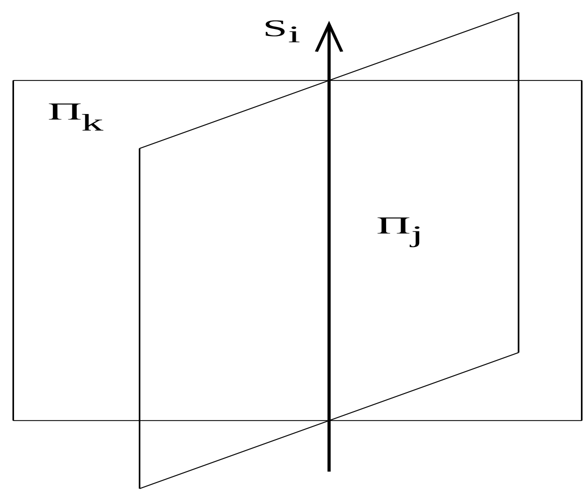

Figure 4. Definition of screw $\mathcal{S}_i$.

where $\mathbf{N}_i$ and $\mathbf{F}_i$ are the local normal and frictional wrenches. f_i and n_i are real numbers representing the intensities of the corresponding wrenches.

Comparing with the previous solution methodology where the external wrenches are mapped into the grasping wrenches $\mathbf{F}_i$, here we would like to map $\mathbf{W}_T$ which is what we call the net wrench to the wrench representation of the friction force at each contact point. Hence it is necessary to define the conditions that can be used to determine the bases of each tangent planes Π_i at local contact points p_i. To accomplish this objective we have defined a number of auxiliary screw representations of the grasp configuration using the local geometry of the grasp.

Let p be the intersection of the tangent planes defined at the local contact points[2]. Let us define screws ξ_i where $\omega_{\xi_i} = u_i$ (equation (1)). u_i is a unit vector defined by vector $\vec{pp_i}$. Let us also define screws Σ_i such that $\Sigma_i(p_j) = \Sigma_i(p_k) = 0$ (e.g. $\Sigma_1(p_2) = \Sigma_1(p_3) = 0$). Another words, the axis of the screw Σ_i is the line $p_j p_k$. The three screws Σ_1, Σ_2 and Σ_3 form a three-system of screws $\mathcal{T}$. It is then always possible to find screws which are in the plane formed by three contact points p_1, p_2, p_3 and also they are in the local tangent planes at each point. Such screws can be obtained as:

$$\eta_i = [\Omega \mathbf{N}_i \mid \Sigma_k]\Sigma_j - [\Omega \mathbf{N}_i \mid \Sigma_j]\Sigma_k$$

At each contact point p_i, ξ_i and η_i are the basis of all the wrenches belonging to the local tangent plane Π_i, i.e. $[\xi_i \mid \eta_i] = 0$. Hence, for a force-closure grasp, any wrench $\mathbf{W}_T$ (equation (12)) can be expanded as a function of the local base screws, or:

$$\mathbf{W}_T = \sum x_k \xi_k + y_k \eta_k \tag{13}$$

here x_k and y_k are the intensities along the base wrenches.

There are many approaches for solving for the intensities of these wrenches. The approach which is used here is to define another set of auxiliary screws which are reciprocal to all but one screw defined by $\{\xi_i, \eta_i\}$.

Let us define the screws $\mathcal{S}_i$ (Figure (4)) which is the intersection of the local tangent planes at contact points p_j and p_k (special singular cases are considered in [5]). It can be shown that the set of screws $\{\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3, \Sigma_1, \Sigma_2, \Sigma_3\}$ and $\{\xi_1, \xi_2, \xi_3, \eta_1, \eta_2, \eta_3\}$ are bi-orthogonal in the sense of inner product space defined in equation (4). For example, referring to Figure (3) and the definition of the basis screws, we can see that screw Σ_1 is reciprocal to screws ξ_3, Σ_2, Σ_3, η_3, ξ_2, η_1 and η_2. Hence, from equation (13) we can solve for intensities:

$$x_i = \frac{[\mathbf{W}_T \mid \Sigma_i]}{[\xi_i \mid \Sigma_i]}$$

Similarly, we can solve for the wrench intensity y_i by multiplying equation (13) by $\mathcal{S}_i$. Hence, we can obtain a solution:

$$y_i = \frac{[\mathbf{W}_T \mid \mathcal{S}_i]}{[\eta_i \mid \mathcal{S}_i]}$$

[2] Special cases where point p can not be located can be found in [5][6]

6 CONCLUSIONS

This paper presented a summary and overview of some of the tools from the algebraic geometry of the screw systems applied to the area of modelling and analysis of multiple contact configuration. It has shown for example how the vector product operation in the space of screws can be used to find a third linearly independent screw. The wrench about this new screw can be used as a location where a third grasping force can be applied on the object. Next, an analytical solution for mapping any external wrench into its wrench components was presented. Finally, a review of a method for obtaining the components of the local friction forces as a function of the external wrench is presented. The geometrical framework of this paper can present itself as an intuitive algebraic geometry approach which can be extended to grasp planning and object manipulation frameworks.

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