# Force and Position Control of Grasp in Multiple Robotic Mechanisms

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**Abstract**

One of the approaches to increase the dexterity of a robot manipulating system is a design philosophy which consists of multiple robotic mechanisms. Applications of such collection of manipulators can be in the design of a dextrous end-effector, a reconfigurable fixture to locate and grip various sized objects or cooperative robotic arms where through their coordinated motions, they are able to accomplish a given task. Although applications of such design philosophy are endless, there are many problems still remain to be addressed. One of these problems is the control of the contact forces (grasping forces) between the mechanisms and the position of the grasped object. This paper addresses this problem. First, a model of the mechanisms in contact with the grasped object is postulated; second the problem of controlling the grasping forces and the position of the grasped object is formulated in the linear multi-input/multi-output system and finally, a centralized optimal controller is proposed for controlling the desired variables. The results of this paper are demonstrated using two examples. One of the main advantages of the proposed optimal controller is that it also shapes the transient response of the grasping force which is an important consideration in cases when grasping fragile objects.

## 1 Introduction

Although robot manipulators have been successfully applied to various tasks, their versatility are limited. For example, when a manipulator is assigned to pick objects with different sizes and attributes, it is usually necessary to change its end-effector with another in order for the manipulator to establish a secure grip. The changing of the end-effectors becomes inevitable when in addition to pick-and-place tasks, the manipulator has to perform a tooling operation (drilling a hole).

In most tasks such as assembly, the picked object has to be placed (inserted) into another one which may have a different size and attributes. In these cases, specialized fixtures have to be designed and integrated with the work-cell of the manipulator in order to hold the object in a fixed position.

One approach which can increase the versatility of a single manipulating mechanism is the utilization of a design philosophy which incorporates a collection of mechanisms. Examples of this design philosophy are: **a)** a dexterous end-effector consisting of a number of open kinematic chains which can be attached to the end-plate of a manipulator, **b)** reconfigurable fixtures consisting of a collection of mechanisms for feeding and holding (grasping) various objects in the work-cell of the manipulator and **c)** two cooperative manipulators equipped with dexterous end-effectors. The last example is an extreme in versatility and autonomy of the automated work-cell. Here, the manipulators can pick-up different sized objects for the purpose of assembly or they can cooperate in handling objects.

There are numerous problems involved in implementing such systems. One of the problems is the control of contacting forces between the manipulators and the object[1,2,3]. Especially, in the case of dextrous end-effectors, it is sometime required to displace the grasped object (grasped object manipulation) so that, for example, it is possible to insert the object into the other,[4,16,6].

The objectives of this paper are: (i) to present a model of a planar grasp between the collection of robotic mechanisms and the object and (ii) to formulate the problem in the state-space domain and to propose a linear optimal centralized controller where the outputs of the system can follow desired values.

The paper is organized into following sections: in section (2) a model of the mechanisms in contact with the grasped object is presented; section (3) gives grasping force and fine-position controller for multiple robotic mechanisms; section (4) demonstrates the results of this paper using two planar mechanisms in contact with the object and finally section (5) presents discussions and outlines the future work.

# 2 A Model of Mechanisms in Contact with the Object

This section presents a model of mechanisms in contact with the object. The model is postulated to be a spring and a damper models. These models are connected in parallel between the mass model of the end-point of the mechanism and the mass model of the object[7]. Figure (1) shows two designs for the end-point of the mechanisms. In both designs, the presence of compliant material (spring and damper models) result in a causal interaction between the end-point of the mechanism and the object. In the design of Figure (1a), due to the nature of contact (point contact with friction) only forces can get transmitted to and from the object)[8].

Let us obtain the linearized model of the finger about an operating point based on the following assumptions: a) we are assuming that the fingers are in contact with the grasped object; b) the configuration parameters of each finger corresponding to its initial contact point is used as the operating point for obtaining the linearized model; c) the dynamic model of the finger corresponding to any deviations from this initial nominal configuration parameters is based on the linearized model.

In a sense what the above standard procedure requires is some *nominal input torques* to the actuators of the fingers which corresponds to the *initial contact configuration* of the finger. The additional torque which is required to move and control the finger from the nominal point is what is developed through the proposed controller. Of course, the issue of how far the linearized model about an operation point is valid is only a function of number of such operating points about the nominal trajectories of the finger.

Let us consider a nonlinear model of a finger:

$$\hat{\tau} = M(\hat{\theta})\hat{\ddot{\theta}} + N(\hat{\theta}, \hat{\dot{\theta}}) + G(\hat{\theta}) + \mathbf{J}(\hat{\theta})^T f_{ext}$$

In the above let the operating point corresponding to the initial condition of a finger be given by $O = (\hat{\tau}, \hat{\theta}, \hat{\dot{\theta}})$. The perturbed model about this initial nominal condition can be written as:

$$\hat{\tau} + \Delta\tau = M(\hat{\theta} + \Delta\theta)(\hat{\ddot{\theta}} + \Delta\ddot{\theta}) + N(\hat{\theta} + \Delta\theta, \hat{\dot{\theta}} + \Delta\dot{\theta}) + G(\hat{\theta} + \Delta\theta) + (\mathbf{J}(\hat{\theta})^T f_{ext} + \mathbf{J}(\hat{\theta} + \Delta\theta)^T f_{ext})$$

Using Taylor series expansion, we have:

$$N(\hat{\theta} + \Delta\theta, \hat{\dot{\theta}} + \Delta\dot{\theta}) = N(\hat{\theta}, \hat{\dot{\theta}}) + \left[\frac{\partial N}{\partial \theta}\right]_O \Delta\theta + \left[\frac{\partial N}{\partial \dot{\theta}}\right]_O \Delta\dot{\theta} + \cdots$$

Or:

$$N(\hat{\theta} + \Delta\theta, \hat{\dot{\theta}} + \Delta\dot{\theta}) = N(\hat{\theta}, \hat{\dot{\theta}}) + \mathbf{C}_1\Delta\theta + \mathbf{C}\Delta\dot{\theta} + \cdots$$

and similarly,

$$G(\hat{\theta} + \Delta\theta) = G(\hat{\theta}) + \left[\frac{\partial G}{\partial \theta}\right]_O \Delta\theta + \cdots = G(\hat{\theta}) + \mathbf{K}\Delta\theta + \cdots$$

Ignoring the second and higher order terms in $\Delta\theta$ and $\Delta\dot{\theta}$ in the above expansions and also assuming $M(\hat{\theta} + \Delta\theta) = M(\hat{\theta})$ (this assumption states that the inertial parameters of the model of the finger has no variation within the bounds from the nominal initial contact point). As a result, the perturbed model of the finger about the nominal point can be written as:

$$\hat{\tau} + \Delta\tau = M(\hat{\theta})(\hat{\ddot{\theta}} + \Delta\ddot{\theta}) + N(\hat{\theta}, \hat{\dot{\theta}}) + \mathbf{C}_1\Delta\theta + \mathbf{C}\Delta\dot{\theta} + G(\hat{\theta}) + \mathbf{K}\Delta\theta + (\mathbf{J}(\hat{\theta})^T f_{ext} + \mathbf{J}(\hat{\theta} + \Delta\theta)^T f_{ext})$$

In view of the initial condition, we have:(substituting the initial condition corresponding to the nominal point into the above equation).

$$M(\hat{\theta})\Delta\ddot{\theta} + \mathbf{C}\Delta\dot{\theta} + (\mathbf{C}_1 + \mathbf{K})\Delta\theta = \Delta\tau - \mathbf{J}(\hat{\theta} + \Delta\theta)^T f_{ext}$$

Or for each finger the above equation can be represented as[10]:

$$\mathbf{M}_{i_{\theta_o}}\ddot{\theta}_i + \mathbf{C}_{i_{\theta_o}}\dot{\theta}_i + \mathbf{K}_{i_{\theta_o}}\theta_i = \tau_{act_i} - \mathbf{J}_i^T f_{ext_i} \tag{1}$$

where, $(\mathbf{M}_{i_{\theta_o}})$ is the inertia matrix of the $i^{th}$ finger, $(\mathbf{C}_{i_{\theta_o}})$ is the linearized velocity dependent terms of the dynamic model and $(\mathbf{K}_{i_{\theta_o}})$ is the position dependent terms such as the torque due to gravity force; $(\tau_{act_i})$ is the actuating torque vector and $(f_{ext_i})$ is a vector of force acting on the $i^{th}$ finger end-point (grasping force vector) which is expressed with respect to its end-point reference coordinate frame[1]. $(\mathbf{J}_i)$ is a linear mapping (Jacobian) from the space of instantaneous properties of the finger end-point to the joint space of each finger. $\theta_i$ is a vector of small joint displacements about the operating point.

Equation (1) can be expressed in the coordinate frame located at the end-point of the manipulator using the following relationship:

$$\begin{aligned} \dot{\theta}_i &= \mathbf{J}_i^{-1}\dot{x}_i \\ \ddot{\theta}_i &= \mathbf{J}_i^{-1}\ddot{x}_i - \mathbf{J}_i^{-1}\dot{\mathbf{J}}_i\mathbf{J}_i^{-1}\dot{x}_i \end{aligned} \tag{2}$$

[1]The actuating force vector is basically the additional torque required when the finger is displaced from its nominal configuration. The remaining of this paper proposes a method for generating this additional torques as a function of the linearized model of the finger.

The dynamic model of each finger expressed with respect to its end-point frame can then be written as[2]:

$$\mathbf{M}_i \ddot{x}_i + \mathbf{C}_i \dot{x}_i + \mathbf{K}_i x_i = f_{act_i} - f_{ext_i} \tag{3}$$

where ($\mathbf{M}_i = \mathbf{J}_i^{-T} \mathbf{M}_{i\theta_o} \mathbf{J}_i^{-1}$) is the coefficient of the inertia matrix expressed in the ($i^{th}$) manipulator end-point frame, ($\mathbf{C}_i = \mathbf{J}_i^{-T} \left[ \mathbf{C}_{i\theta_o} - \mathbf{M}_{i\theta_o} \mathbf{J}_i^{-1} \dot{\mathbf{J}}_i \right] \mathbf{J}_i^{-1}$) is the coefficient of force due to the velocity dependent term and ($\mathbf{K}_i = \mathbf{J}_i^{-T} \mathbf{K}_{i\theta_o}$) is the coefficient of force due to the position dependent terms. In the case when the plane of grasping and manipulation is perpendicular to the direction of the gravity, in equation (3), the effect of position dependent forces due to gravity is ignored.

Let us assume that planar dynamic model of the object expressed with respect to its coordinate frame can also be written as:

$$\sum_{i=1}^{n} f_{ext_i} = \mathbf{M}_o \ddot{x}_o \tag{4}$$

in the above equation ($f_{ext_i}$) is the grasping force of the ($i^{th}$) mechanism where its components are expressed with respect to the reference coordinate frame of the object and $\ddot{x}_o$ is the linear acceleration vector of the object. The above model is not the restriction but rather is used to facilitate the demonstration of the performance of the controller which is shown through-out the rest of the paper (the model of the grasped object can be viewed as a point mass). This model assumes that the motion of the the grasped to have only translational components with respect to its coordinate frame and also ignores the effect of gravity acting on the grasped object.

Based on the above motivation and procedures we are assuming that the fingers have established the initial contact with the grasped object (i.e. the fingers move in an exploratory procedures until they detect contact with the object through force sensing modality). At this stage and assuming that some disturbances due to the impact with the object have been vanished, the proposed controller of the paper trys to regulate the grasping force and position. Although the procedure of linearization of this paper is general, but we are proposing that the nominal point corresponds to zero velocity of the fingers.

# 3 Force and Position Controller

The objective of this section is to propose a controller which can regulate both the contact forces (grasping forces) that each mechanisms exerts on the object and their corresponding end-point positions.

In the following, the grasp configuration is a planar one where the plane of grasp is perpendicular to the direction of the gravity forces. Also, given the model of the compliant finger-tips, the grasping forces are functions of the mechanisms and object displacements and the model of the material of the compliant-tip, or:

$$f_{ext_i} = f(x_i, \dot{x}_i, x_o, \dot{x}_o, \mathbf{K}_{c_i}, \mathbf{C}_{c_i}) \qquad 1 \leq i < n \tag{5}$$

where $\mathbf{K}_{c_i}$ and $\mathbf{C}_{c_i}$ are models of spring and damping properties of the compliant finger-tip and $x_o$ and $\dot{x}_o$ are the position and velocity of the grasped object. For example, the external grasping force

[2]It is assumed that in grasping and at the initial instance of contact with the object, the fingers have zero initial velocity.

acting on the $i^{th}$ finger can be written as:

$$f_{ext_i} = \mathbf{K}_{c_i}(x_o - x_i) + \mathbf{C}_{c_i}(\dot{x}_o - \dot{x}_i)$$

As a result, the end-point compliance of each finger couples the dynamics of the finger with the grasped object. This is accomplished by noting that the grasping force between each finger and the object is a function of the state variable of the finger and that of the object mapped through the model of the compliance. Expressing linearized dynamic model of each finger and the grasped object (equations 3 and 4) by taking into account the coupling factor of equation (5) we have:

$$\begin{cases} \mathbf{M}_1\ddot{x}_1 + (\mathbf{C}_1 + \mathbf{C}_{c_1})\dot{x}_1 + \mathbf{K}_{c_1}x_1 - \mathbf{C}_{c_1}\dot{x}_o - \mathbf{K}_{c,1}x_o = f_{act_1} \\ \vdots \\ \mathbf{M}_i\ddot{x}_i + (\mathbf{C}_i + \mathbf{C}_{c_i})\dot{x}_i + \mathbf{K}_{c_i}x_i - \mathbf{C}_{c_i}\dot{x}_o - \mathbf{K}_{c,i}x_o = f_{act_i} \\ \vdots \\ \mathbf{M}_o\ddot{x}_o + (\mathbf{C}_{c_1} + \mathbf{C}_{c_2} + \cdots)\dot{x}_o - \mathbf{C}_{c_1}\dot{x}_1 - \mathbf{C}_{c_2}\dot{x}_2 - \cdots \\ (\mathbf{K}_{c_1} + \mathbf{K}_{c_2} + \cdots)x_o - \mathbf{K}_{c_1}x_1 - \mathbf{K}_{c_2}x_2 - \cdots = 0 \end{cases} \tag{6}$$

The above equation can be transformed into the standard state-space model as:

$$\dot{x} = \mathbf{A}x + \mathbf{B}u \tag{7}$$

where $(x = (x_1, x_2, \cdots, x_i, x_o, \dot{x}_1, \dot{x}_2, \cdots, \dot{x}_i, \dot{x}_o)^T) \in \mathbf{R^n}$ is the state vector of the compounded finger/object system and $(u = (f_{act_1}^T, f_{act_2}^T, \cdots, f_{act_i}^T)^T) \in \mathbf{R^r}$ is the actuating force vector.

The output vector of the system can be written as:

$$y = \begin{Bmatrix} y_1 \\ y_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{C}_1 \\ \mathbf{C}_2 \end{bmatrix} x \tag{8}$$

where $(y)$ is a vector composed of grasping forces, the end-point positions of the fingers and the position of the grasped object.

Given the dynamic model of the fingers and object, the objective of the controller is for the outputs $y_1 \subset y$ of the system to track some desired reference constant (or periodically changing) input vector $(y_r \in \mathbf{R}^p)$ containing the desired grasping forces and positions of fingers[9]. Defining the error between the desired reference and the actual one as:

$$\dot{z} = y_r - y_1 = y_r - \mathbf{C}_1 x \tag{9}$$

and augmenting equation (7) with (8) results in $(n + p)$ dimensional system:

$$\begin{aligned} \dot{\bar{x}} &= \bar{\mathbf{A}}\bar{x} + \bar{\mathbf{B}}u + \mathbf{D}y_r \\ \bar{y} &= \bar{\mathbf{C}}\bar{x} \end{aligned} \tag{10}$$

where

$$\bar{x} = \begin{bmatrix} x \\ z \end{bmatrix}, \quad \bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C}_1 & \mathbf{0} \end{bmatrix}, \quad \bar{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}, \quad \bar{\mathbf{C}} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

Consider now differentiating (10) to get

$$\ddot{\bar{x}} = \bar{\mathbf{A}}\dot{\bar{x}} + \bar{\mathbf{B}}\dot{u} \tag{11}$$

It is now desired to obtain the proportional-integral (PI) optimal control law ($u$) of the form

$$u = \mathbf{K}_1 x + \mathbf{K}_2 \int z dt \tag{12}$$

such that the following performance measure is minimized subject to (11)

$$J = \frac{1}{2}\int_0^\infty (\|\dot{\bar{x}}\|^2{}_{\mathbf{Q}} + \|\dot{u}\|^2{}_{\mathbf{R}})dt \tag{13}$$

and the closed-loop eigenspectrum of the system are assigned to desired locations. As a result, the transient response of the output can be shaped such that the force response do not have any large over-shoots.

Using either of the sequential approach as described in[11,12], it can be shown that for a fixed $\mathbf{R}$, if $\mathbf{Q}$ in (12) is appropriately selected, the optimal control law that would achieve the above objectives would be given by:

$$u = -\mathbf{R}^{-1}\bar{\mathbf{B}}^T\mathbf{P}\bar{x} = \mathbf{K}\bar{x} \tag{14}$$

where $\mathbf{P}$ is the positive semidefinite solution of the Algebraic Matrix Riccati Equation (AMRE). Various necessary and sufficient conditions for existence of a solution to the above problem can be found in[11]. It can also be shown[13] [13] that the pair $\left\{\bar{\mathbf{A}}, \bar{\mathbf{B}}\right\}$ is completely controllable if and only if $\{\mathbf{A}, \mathbf{B}\}$ is controllable and,

$$\mathbf{\Omega} = \begin{bmatrix} \mathbf{B} & \mathbf{A} \\ \mathbf{0} & -\mathbf{C} \end{bmatrix}$$

has rank $(n + p)$, where $(p)$ is the number of outputs that need to regulated.

The summary of the sequential approach as proposed in[12] is as follows: suppose that at the $ith$ stage of the sequential process the system under consideration described by[3]:

$$\ddot{\bar{x}}_i = \bar{\mathbf{A}}_i\dot{\bar{x}}_i + \bar{\mathbf{B}}\dot{u}_i \tag{15}$$

is aggregated ( reduced ) to an $lth$ order dynamical system, where $l = 1$, or 2 depending on whether a real, or a complex conjugate pair of pole(s) is being placed. The reduced order system is described by:

$$\ddot{\hat{x}}_i = \hat{\mathbf{A}}_i\dot{\hat{x}}_i + \hat{\mathbf{B}}_i\dot{\hat{u}}_i \tag{16}$$

where $l$ of the eigenvalues of $\bar{\mathbf{A}}_i$ in (15) are contained in $\hat{\mathbf{A}}_i$, that is $\Lambda(\hat{\mathbf{A}}_i) \subset \Lambda(\bar{\mathbf{A}}_i)$, where $\Lambda(.)$ is a set that contains the eigenvalues of the matrix in the argument. Note that from here on the variables with $\hat{()}$ will refer to the reduce order system. The aggregation matrix which would accomplish the above transformation is given by:

$$\mathbf{\Gamma} = [\mathbf{I}_l \quad \mathbf{0}\ ]\mathbf{\Phi}^{-1}(\bar{\mathbf{A}}_i)$$

where $\mathbf{\Phi}(.)$ is the modal matrix of the matrix argument, and in this case its first $l$ columns are the eigenvectors corresponding to the $l$ eigenvalues of $\hat{\mathbf{A}}_i$. The matrices $\hat{\mathbf{A}}_i$ and $\hat{\mathbf{B}}_i$ are obtained by using the following transformations:

$$\begin{array}{lll} \hat{\mathbf{A}}_i = \mathbf{\Gamma}\bar{\mathbf{A}}_i\mathbf{\Gamma}^+ & \hat{\mathbf{B}}_i = \mathbf{\Gamma}\bar{\mathbf{B}} & \text{if l=1} \\ \hat{\mathbf{A}}_i = \mathbf{\Theta}^{-1}\mathbf{\Gamma}\bar{\mathbf{A}}_i\mathbf{\Gamma}^+\mathbf{\Theta} & \hat{\mathbf{B}}_i = \mathbf{\Theta}^{-1}\mathbf{\Gamma}\bar{\mathbf{B}} & \text{if l=2} \\ \mathbf{\Theta} = \begin{bmatrix} 0.5 & +j0.5 \\ 0.5 & -j0.5 \end{bmatrix} & & \end{array}$$

[3]The sequential process starts at stage $i = 1$, where $\bar{\mathbf{A}}_1 = \bar{\mathbf{A}}$, and $\bar{\mathbf{A}}$ is given as in (10).

where $\mathbf{\Gamma}^+$ is the Moore-Penrose pseudo inverse of $\mathbf{\Gamma}$ given by $\mathbf{\Gamma}^+ = \mathbf{\Gamma}^T(\mathbf{\Gamma}\mathbf{\Gamma}^T)^{-1}$. It is now desired to obtain a desired state weighting matrix ($\hat{\mathbf{Q}}_i$) in the following quadratic cost function:

$$\hat{\mathbf{J}}_i = \frac{1}{2}\int_o^\infty \left(\|\dot{\hat{x}}_i\|^2_{\hat{\mathbf{Q}}_i} + \|\dot{\hat{u}}_i\|^2_{\mathbf{R}}\right) dt \tag{17}$$

so that the optimal control law which would minimize (17) would place the real or the complex conjugate poles of the aggregated system ($\hat{\mathbf{A}}_i$) at desired location. This optimal control law is given by:

$$\dot{\hat{u}}_i = -\mathbf{R}^{-1}\hat{\mathbf{B}}_i^T\hat{\mathbf{P}}_i\dot{\hat{x}}_i = \hat{\mathbf{K}}_i\dot{\hat{x}}_i \tag{18}$$

where $\hat{\mathbf{P}}_i$ is the solution to the following AMRE:

$$\hat{\mathbf{P}}_i\hat{\mathbf{A}}_i + \hat{\mathbf{A}}_i^T\hat{\mathbf{P}}_i - \hat{\mathbf{P}}_i\hat{\mathbf{B}}_i\mathbf{R}^{-1}\hat{\mathbf{B}}_i^T\hat{\mathbf{P}}_i + \hat{\mathbf{Q}}_i = \mathbf{0} \tag{19}$$

Using the above control law, the closed loop reduced order system would be:

$$\ddot{\hat{x}}_i = (\hat{\mathbf{A}}_i + \hat{\mathbf{B}}_i\hat{\mathbf{K}}_i)\dot{\hat{x}}_i = \hat{\mathbf{A}}_{ci}\dot{\hat{x}}_i$$

Now assuming that the system's eigenvalues are distinct and a desired set of closed loop eigenspectrum is given, it is easy to show that (19) can be written as:

$$\mathbf{\Phi}^T(\hat{\mathbf{A}}_{ci})\hat{\mathbf{P}}_i\mathbf{\Phi}(\hat{\mathbf{A}}_{ci})\mathbf{\Sigma}_i + \mathbf{\Sigma}_i\mathbf{\Phi}^T(\hat{\mathbf{A}}_{ci})\hat{\mathbf{P}}_i\mathbf{\Phi}(\hat{\mathbf{A}}_{ci}) = -\mathbf{\Phi}^T(\hat{\mathbf{A}}_{ci})[\hat{\mathbf{Q}}_i + \hat{\mathbf{K}}_i^T\mathbf{R}\hat{\mathbf{K}}_i]\mathbf{\Phi}(\hat{\mathbf{A}}_{ci})$$

where $\mathbf{\Phi}(.)$ is as defined earlier, and $\mathbf{\Sigma}_i$ is a diagonal matrix which has the desired closed loop eigenspectrum on its main diagonal.

It can be shown that by selecting $\hat{\mathbf{Q}}_i$ as:

$$\hat{\mathbf{Q}}_i = [\mathbf{\Phi}(\hat{\mathbf{A}}_{ci})\mathbf{\Phi}^T(\hat{\mathbf{A}}_{ci})]^{-1} - \hat{\mathbf{K}}_i^T\mathbf{R}\hat{\mathbf{K}}_i \tag{20}$$

the solution of AMRE can be obtained without a need to solve (19), and hence, the optimal feedback gain in (18) would be given by:

$$\hat{\mathbf{K}}_i = \frac{1}{2}\mathbf{R}^{-1}\hat{\mathbf{B}}_i^T\mathbf{\Phi}^{-T}(\hat{\mathbf{A}}_{ci})\mathbf{\Sigma}_i^{-1}\mathbf{\Phi}^{-1}(\hat{\mathbf{A}}_{ci}) \tag{21}$$

Note that in the above equation the value of matrix $\mathbf{\Phi}(.)$ is as yet unknown. The following relationship between the eigenvalues and eigenvectors of a matrix would provide additional information for computing the feedback gain:

$$\begin{aligned} &\hat{\mathbf{A}}_{ci}v_l = \lambda v_l \qquad\qquad \text{for l=1,2} \quad \textit{or} \\ &[(\hat{\mathbf{A}}_i - \lambda_l\mathbf{I}) \quad \hat{\mathbf{B}}_i\ ]\begin{Bmatrix} v_l \\ \psi_l \end{Bmatrix} = 0 \end{aligned} \tag{22}$$

It is easy to show[14] now that the feedback gain $\hat{\mathbf{K}}_i$ is given by:

$$\hat{\mathbf{K}}_i = \mathbf{\Psi}\mathbf{\Phi}^{-1}(\hat{\mathbf{A}}_{ci}) \tag{23}$$

where $\mathbf{\Psi} \in \mathbf{R}^{q\times l}$ is a matrix whose columns are given by $\{\psi_l = \hat{\mathbf{K}}_i v_l\}$ for $l = 1, 2$. Finally, equating (21) and (23) will result in:

$$\mathbf{\Phi}(\hat{\mathbf{A}}_{ci})\mathbf{\Sigma}_i\Psi^T = \frac{1}{2}\hat{\mathbf{B}}_i\mathbf{R}^{-1} \tag{24}$$

Equations (22) and (25) are now solved together to obtain $\mathbf{\Phi}(.)$ and $\mathbf{\Psi}$. Then, the optimal feedback gain and the desired weighting are obtained from (21) and (20) respectively. Once again, it should be noted that all of the above computations involve scalar or second order systems, and thus the computational requirements is extremely light.

Once the appropriate matrices $\hat{\mathbf{Q}}_i$ and $\hat{\mathbf{K}}_i$ in (17) and (18) are found, they are transformed back to the original higher dimensional space via the following transformations to obtain $\mathbf{Q}_i$, and $\mathbf{K}_i$ that would assign the same poles for the system (11).

$$\begin{array}{ll} \mathbf{K}_i = \hat{\mathbf{K}}_i\mathbf{\Gamma} & \text{if l=1} \\ \mathbf{Q}_i = \mathbf{\Gamma}^T\hat{\mathbf{Q}}_i\mathbf{\Gamma} & \\ \mathbf{K}_i = \hat{\mathbf{K}}_i\mathbf{\Theta}^{-1}\mathbf{\Gamma} & \text{if l=2} \\ \mathbf{Q}_i = \mathbf{\Gamma}^T\mathbf{\Theta}^{-T}\hat{\mathbf{Q}}_i\mathbf{\Theta}^{-T}\mathbf{\Gamma} & \end{array}$$

Next the system dynamics will be updated by:

$$\bar{\mathbf{A}}_{i+1} = \bar{\mathbf{A}}_i + \bar{\mathbf{B}}\mathbf{K}_i$$

The above describes one stage of the sequential procedure. Now letting $i = i + 1$, equation (11) will be aggregated to a first or second order system for placing another real or complex conjugate pair of pole(s). The sequential process will continue in this manner until all or a number of the dominant poles of the system are placed. At this time the overall desired weighting matrix $\mathbf{Q}$ in (13) and the optimal state feedback gain $\mathbf{K}$ in (14), that would achieve the pole placement will be calculated from:

$$\mathbf{Q} = \textstyle\sum_i \mathbf{Q}_i$$
$$\mathbf{K} = \textstyle\sum_i \mathbf{K}_i$$

This completes the servomechanism design procedure. The above feedback gain, the weighting matrix and the control law in (14) will minimize the cost function in (13). The closed loop system will now be described by:

$$\dot{\bar{x}} = (\bar{\mathbf{A}} + \bar{\mathbf{B}}\mathbf{K})\bar{x} + \bar{\mathbf{D}}y_r$$

where the eigenvalues of $(\bar{\mathbf{A}} + \bar{\mathbf{B}}\mathbf{K})$ are at the desired locations. Also, the output $y_1$ will now track the reference input $y_r$.

# 4 Example

This section presents the performance of the optimal controller using two examples. The first example consists of two 1DOF fingers grasping an object. Each finger-tip is assumed to be constructed with known compliant material (see Figure (2)). In addition, the velocity dependent terms in the dynamic model of each finger are neglected ($\mathbf{C}_i$ in equation (3)).

The state-space model of the system can be written as:

$$\begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{k_c}{m_{f_1}} & 0 & \frac{k_c}{m_{f_1}} & -\frac{c_c}{m_{f_1}} & 0 & \frac{c_c}{m_{f_1}} \\ 0 & -\frac{k_c}{m_{f_2}} & \frac{k_c}{m_{f_2}} & 0 & -\frac{c_c}{m_{f_2}} & \frac{c_c}{m_{f_2}} \\ \frac{k_c}{m_o} & \frac{k_c}{m_o} & -\frac{2k_c}{m_o} & \frac{c_c}{m_o} & \frac{c_c}{m_o} & -\frac{2c_c}{m_o} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_{f_1}} & 0 \\ 0 & -\frac{1}{m_{f_2}} \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} f_{act_1} \\ f_{act_2} \end{Bmatrix} \quad (25)$$

The measured output of the system can be written as:

$$\left\{ \begin{array}{c} f_{ext_1} \\ f_{ext_2} \\ x_{f_1} \\ x_{f_2} \end{array} \right\} = \left[ \begin{array}{cccccc} k_c & 0 & -k_c & c_c & 0 & -c_c \\ 0 & k_c & -k_c & 0 & c_c & -c_c \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right] \left\{ \begin{array}{c} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{array} \right\} \tag{26}$$

where $(x_1, x_2, x_3)$ are the positions of the fingers and object and $(x_4, x_5, x_6)$ are the corresponding velocities.

In the above equation, the parameters of the system are given as: $m_{f_1} = m_{f_2} = 0.4(Kg)$ ;$m_o = 0.2(Kg)$; $k_c = 100(N/m)$; $c_c = 100(N/(m/sec))$. The objective is for each finger to exert (1 Newton) force on the grasped object while finger #1 moves (.5 cm ) from its reference position.

To arrive at the tracking controller, the augmented system in (9) was formed. It can be verified that the open loop poles of the augmented system are given by { ( 0,0,-1.25,-0.001, 0,-0.249,-0.001, 0) $\star$ 1E03 }. Since two of the open loop poles far in the left hand plane, it was decided not to spend any control effort to move these non-dominant modes. Also, we decided to preserve the two poles at -1.0. The remaining four poles of the system are to be assign to $\{-1.5, -2.0, -2.5, \text{and } -3.0\}$. In addition, since all of the state variables of the system are not available for measurements and feedback, an estimator is to be designed as well. The design of the estimator require[15] the observation matrix $\bar{\mathbf{C}}$ in (9) be of the form:

$$\bar{\mathbf{C}} = \left[ \begin{array}{cc} \mathbf{I} & \mathbf{0} \end{array} \right]$$

Since this is not the case in the example, the system in (9) was transformed via the following similarity transformation matrix, to get the resulting observation matrix in the desired form given above,

$$\mathbf{T} = \left[ \begin{array}{rrrrrrrr} 100 & 0 & -100 & 100 & 0 & -100 & 0 & 0 \\ 0 & 100 & -100 & 0 & 100 & -100 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 10 & 30 & 0 & 0 & 0 & 30 & 0 & 70 \\ 0 & 0 & 0 & 50 & 10 & 0 & 30 & 0 \end{array} \right]$$

Once this was accomplished a reduced ( second order ) estimator with eigenvalues at $\{-10, -8\}$ was designed for the transformed system. The dynamics of the estimator is given by:

$$\dot{\omega} = \mathbf{F}\omega + \mathbf{G}y + \mathbf{H}u + \mathbf{S}y_r$$

where:

$$\mathbf{F} = \left[ \begin{array}{rr} -10. & 0 \\ 0 & -8 \end{array} \right] \quad ; \quad \mathbf{G} = 1E03 \star \left[ \begin{array}{rrrrrr} -1.35 & -2.07 & 0.02 & -3.09 & 0 & 0.7 \\ -0.151 & -0.029 & -0.4 & -3.44 & 0.24 & 0 \end{array} \right]$$

$$\mathbf{H} = \left[ \begin{array}{rr} 0 & -750 \\ 125 & -25 \end{array} \right] \quad ; \quad \mathbf{S} = \left[ \begin{array}{rr} 0 & 70 \\ 30 & 0 \end{array} \right]$$

and the estimate of the missing state variable of the transformed system are given by:

$$\omega + \mathbf{M}y$$

where:

$$\mathbf{M} = \begin{bmatrix} 0 & -3 & 0 & 340 & 0 & 0 \\ 0 & 0 & 0 & 480 & 0 & 0 \end{bmatrix}$$

Note that the sequential optimal weight selection procedures described in[11], 12 do not necessarily result in unique weights or optimal controller gains. One set of optimal gains and corresponding state weighting matrices in the cost function were calculated. Once these values were obtained, they were transformed back using the transformation $\mathbf{T}$, to get the following sequential optimal feedback gain matrices:

$$\mathbf{K}_1 = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.3000 & 0.3000 & 0.1500 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & -0.3000 & -0.3000 & -0.1500 & 0.0000 & 0.0000 \end{bmatrix}$$

$$\mathbf{K}_2 = \begin{bmatrix} 0.0000 & 0.0000 & 0.0000 & 0.4000 & -0.4000 & -0.2000 & -2.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.4000 & -0.4000 & -0.2000 & -2.0000 & 0.0000 \end{bmatrix}$$

$$\mathbf{K}_3 = \begin{bmatrix} 0.7500 & 0.7500 & 0.3750 & 0.5000 & 0.5000 & 0.2500 & 0.0000 & 0.0000 \\ -0.7500 & -0.7500 & -0.3750 & -0.5000 & -0.5000 & -0.2500 & 0.0000 & 0.0000 \end{bmatrix}$$

$$\mathbf{K}_4 = \begin{bmatrix} 5.7749 & 0.1500 & 0.0750 & 0.6000 & 0.6000 & 0.3045 & 0.0563 & -5.6249 \\ -5.7750 & -0.1500 & -0.0750 & -0.6000 & -0.6000 & -0.3045 & -0.0563 & 5.6250 \end{bmatrix}$$

where we have:

$$\mathbf{K} = \Sigma_{i=1}^{4}\mathbf{K}_i = \begin{bmatrix} 6.525 & 0.9 & 0.45 & 1.8 & 1. & 0.504 & -1.943 & -5.624 \\ -6.525 & -0.9 & -0.45 & -1. & -1.8 & -.904 & -2.05 & 5.625 \end{bmatrix}$$

The sum of transformed sequential state weighting matrix $\mathbf{Q}_i$ which is obtained through the relationship $\mathbf{Q}_i = \mathbf{\Gamma^T \hat{Q}_i \Gamma}$ is calculated to be:

$$\mathbf{Q} = \Sigma_{i=1}^{4}\mathbf{Q}_i = \begin{bmatrix} 33.91 & 1.428 & 0.7144 & 3.84 & 3.84 & 1.94 & 0.324 & -32.48 \\ 1.428 & 0.585 & 0.292 & 0.465 & 0.465 & 0.233 & 0.008 & -.843 \\ 0.714 & 0.292 & 0.146 & 0.232 & 0.232 & 0.116 & 0.004 & -0.421 \\ 3.84 & 0.465 & 0.232 & 0.86 & 0.54 & 0.272 & -0.766 & -3.375 \\ 3.84 & 0.465 & 0.232 & 0.54 & 0.86 & 0.432 & 0.833 & -3.375 \\ 1.946 & 0.232 & 0.116 & 0.272 & 0.432 & 0.217 & 0.417 & -1.712 \\ 0.324 & 0.008 & 0.004 & -0.766 & 0.833 & 0.417 & 4.003 & -0.316 \\ -32.483 & -0.843 & -0.421 & -3.375 & -3.375 & -1.712 & -0.316 & 31.639 \end{bmatrix}$$

It can be verified that application of optimal control law in (12) with $\mathbf{K}$ given in the above will achieve the desired closed loop poles and at the same time minimizes the cost function given in equation (13) with poles defined as above.

For command inputs of $1.0N$ in the desired grasping force and $0.5cm$ in displacement of finger #1, Figure (2) shows the response of the optimal controller/estimator system. Clearly the controller performance as desired where for example, finger #2 follows the desired displacement of finger #1.

In the second example, each finger is modelled to have two rotary joints with the dimension of each link given as ($l_1 = 0.153m$ and $l_2 = 0.123m$) and mass of ($m_1 = 0.053Kg$ and $m_2 = 0.02Kg$) (similar to the model used by[16]) (Figure 4). The initial configuration parameters of the fingers are $\theta_{1_1} = 135°, \theta_{1_2} = 45°, \theta_{2_1} = -90°, \theta_{2_2} = 90°$.

The compliant material of both finger-tips are modeled as linear springs having stiffness of ($1000N/m$). The mass of the grasped object $m_0 = 0.2Kg$. The objective of the simulation is for each finger to exert forces equal to 5 and then 2 Newton on the grasped object and moving the object 1 and then 2 cm from its initial grasp configuration. Since the proposed controller of equation (9) requires full state feedback of the finger/object system, an observer is designed[15] for estimating the state of the object (i.e. it is assumed that the position and velocity of the grasped object are not measurable). Following the procedure for designing the optimal controller outlined in the previous section, gains of the controller were selected such that the closed-loop response has minimum overshoot. Figures 5 and 6 shows responses of the controller for step changes in the desired grasping forces and the position of the grasped object.

# 5 Conclusions and Future Work

This paper presents a model of robotics mechanisms (fingers) in contact with the grasped object. In this model compliant material is attached between the end-points of the mechanisms and the object. Using this model, a centralized force and position controller is proposed. The main feature of this controller is that it minimizes a performance index while it also places the closed loop poles of the system at desired locations. This feature of the controller is important in multiple mechanical fingers grasping a fragile object where the objective is to shape the transient response of the system. The results of this paper is demonstrated using two examples.

A number of issues remain that need to be addressed: 1) experimental validation of the responses proposed controller (an experimental set-up consisting of two 2DOF fingers are begin developed); 3) development of Decentralized optimal controller and its comparison with the controller proposed in this paper; 3) extension of the proposed controller for controlling force and position of *power grasp*[17]).

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*Figure 1: Two designs for the tips of mechanisms.*

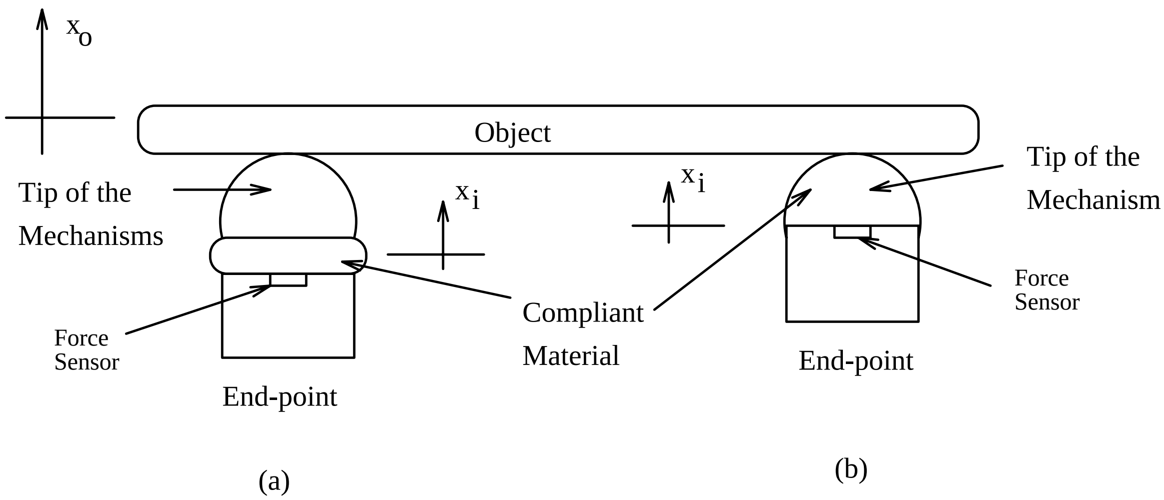
*Figure 2: Two fingers grasping an object.*

*Figure 3: The responses of finger #1 and #2 to the input grasping force and displacement. Solid lines are grasping force responses of the fingers to 1 Newton for the desired values and the dashed lines are the fingers displacements.*

*Figure 4: The schematics of the second example.*

*Figure 5: The force responses of the grasping fingers. Solid line is the left finger and the dashed line is the right.*

*Figure 6: The position response of the grasped object.*

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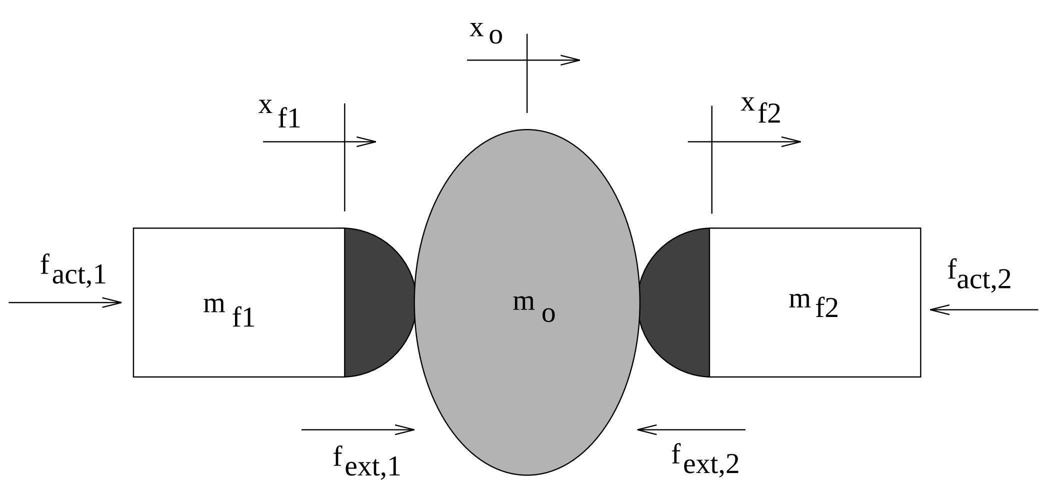


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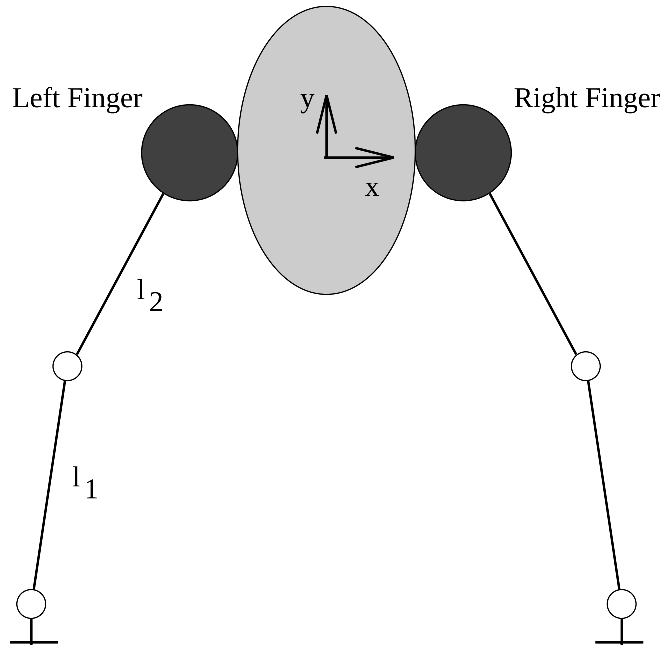


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