On Invariant Configuration of a Three-Fingered Grasp

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Abstract

When grasping an object using three fingered end-effectors, a certain geometrical entity can be created based on the grasping points and a definition of an auxiliary point. This geometric configuration is referred to as the invariant configuration of the three fingered grasp. This paper exploits the geometric properties of such configuration using screw theory and inner product spaces. The results are shown to present themselves as an elegant and efficient framework in calculating the friction forces between the fingers and the grasped object and/or in defining the instantaneous motion of the finger-tips in accomplishing the desired motion of the grasped object.

1 Introduction

When grasping objects with a dexterous mechanical end-effector, the contact between the fingers of the hand and the object must satisfy a number of conditions[1,2,3]. These range from the force-closure to stability conditions such as, ensuring that the grasped object does not slip between the fingers. Manipulation of the grasped object is further defined as the ability of the mechanical end-effector to create an instantaneous motion of the grasped object with respect to a fixed reference frame (e.g. palm reference frame).

Authors of[4] were among the first to consider the problem of computing the friction forces between the finger-tips and the grasped object as a function of the external forces. Their approach takes advantage of the grasp configuration and the screw geometry. However, their method did not take advantage of the fundamental coordinate free property of screw systems and it was rather an algebraic approach. Various methods have been proposed in the literature for determining the motion of the grasping fingers in order to create the desired motion of the grasped object. Several results on the existence of a certain three to four-fingered grasp being suitable for fine manipulation when given some constraints on the geometry of the object was presented by[5]. A method for decomposing the finger-tip force into two components by using pseudo-inverse of coefficient matrix that relates the resultant force exerted on the grasped object to the fingertip force was presented by[6]. The first decomposed component is referred to as the manipulation and the other as the grasping forces. A strategy for dexterous manipulation called finger tracking was proposed by[7]. The reorientation of the grasped object is planned by finger tracking on the face of an object whose motion is constrained by a set of fingers fixed in space. In this way, the re-orientation is accomplished by simple sliding motion of the tracking finger. A method where manipulation of the grasped object can be accomplished by sliding action of one finger on the face of the object was suggested by[8]. The sliding trajectories are characterized by a transient and steady-state solution. By classifying the configuration of fingers grasping an object as a configuration of parallel chain manipulator, a method based on screw system for obtaining transformation equations between joint coordinates of the fingers and the corresponding finger-tip displacements was proposed by[9].

This paper presents an elegant and efficient framework which can be used for calculating the friction forces as a function of the external force acting on the grasped object and the instantaneous properties of the finger-tips as a function of the motion of the grasped object. The method improvises the geometric properties of the grasp object and the grasp configuration to define a new geometrical entity. Then based on screw geometry and inner product spaces, an elegant and efficient approaches are presented to map the external force into the required friction forces and the desired grasped object motion into the corresponding finger-tip motions.

This paper is organized as follows: section (2) presents some preliminaries definitions; section (3) presents the main results of the paper and section (4) presents a sample example and finally section (5) presents concluding remarks and future work.

2 Preliminaries

Let $\mathbf{X}$ define a vector field (*helicoidal field*) $\mathbf{X} : \varepsilon \rightarrow \mathcal{E}$ such that (Figure (1)):

$$\mathbf{X}(b) = \mathbf{X}(a) + \omega_{\mathbf{X}} \times \vec{ab} \tag{1}$$

where a and b are any two points belonging to the tridimensional affine Euclidean space ε and $\mathcal{E}$ is the underlying vector space[10].

This is equivalent to say that for every pair of points (a, b) in ε, there exists a vector $\omega_{\mathbf{X}}$ such that equation (1) holds. Let $\mathcal{D}$ represent such space.

For example, when the velocity of a rigid body is described by the twist $T = (\omega; u)$, the helicoidal velocity field V, is related to T by:

$$V(p) = u + \omega \times \vec{op} \tag{2}$$

where p is any point in ε. For a wrench $W = (f; g)$, the associated helicoidal field M (that is the moment field) is defined as:

$$M(p) = g + f \times \vec{op} \tag{3}$$

For $\mathbf{X}$, $\mathbf{Y} \in \mathcal{D}$, let us define the real number:

$$[\mathbf{X} \mid \mathbf{Y}] = \omega_{\mathbf{X}} \cdot \mathbf{Y}(p) + \omega_{\mathbf{Y}} \cdot \mathbf{X}(p) \tag{4}$$

In general, the above relationship is independent of the choice of p in ε (The above operation is also referred to as the inner product or the Klein form of screws. We use the word *inner product* even though the Klein form is not positive definite. It should be pointed-out that no positive definite inner product exists in screw theory which is invariant by the Euclidean group). For example,

$$[W \mid T] = u \cdot f + \omega \cdot g \qquad \text{a real number}$$

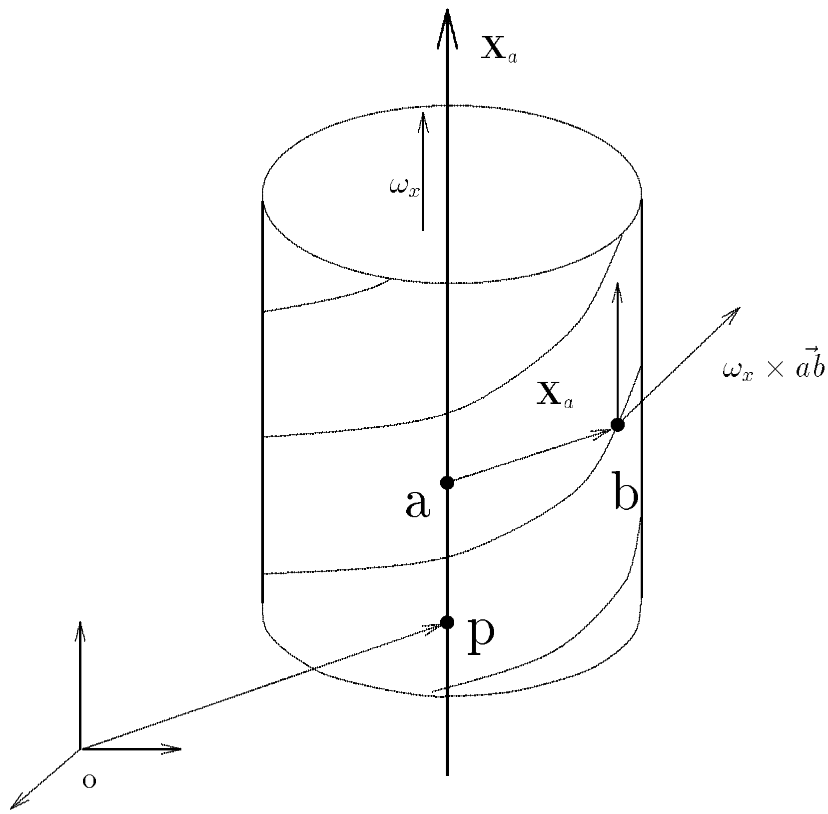

Figure 1: General definition of helicoidal field

The relationship $[\mathbf{X} \mid \mathbf{Y}] = 0$ means that the screws are reciprocal.

We further define operation Ω. If $\mathbf{X} \in \mathcal{D}$, then $\Omega\mathbf{X}$ is the constant vector field given by:

$$\Omega\mathbf{X}(p) = \omega_{\mathbf{X}} \qquad \text{for all } p \in \epsilon \tag{5}$$

Hence $\Omega^2 = 0$.

2.1 Definition of Three-Fingered Grasp

We define three-fingered grasp by three points (p_1, p_2, p_3) specifying the location of the finger/contact coordinate systems and the direction of the normals to the surface of the object at the contact points.

Along each surface normals, we define normalized screws ν_1, ν_2 and ν_3, where for $i = 1, 2, 3$ we have:

$$\begin{array}{l} [\nu_i \mid \Omega\nu_i] = 1 \\ p_i \in \text{axis of } \nu_i \end{array} \tag{6}$$

The screw ν_i describes the oriented normal to the surface of the grasped object at the point p_i. These screws are defined by screws with zero pitch.

Let Π_i denote the tangential plane at p_i. Let $\mathcal{Z}_i$ be the vector subspace of $\mathcal{D}$ which is defined as:

$$\mathcal{Z}_i = \{\mathbf{X} \in \mathcal{D} \mid \mathbf{X}(p_i) = 0\} \tag{7}$$

and let $\mathcal{F}_i$ be the vector subspace of $\mathcal{Z}_i$ defined by:

$$\mathcal{F}_i = \{\mathbf{X} \in \mathcal{D} \mid \mathbf{X}(p_i) = 0, [\mathbf{X} \mid \Omega\nu_i] = 0\}$$

where $\nu_i \in \mathcal{Z}_i$ (Figure 2). For example, for a finger-tip twist T belonging to $\mathcal{Z}_i$, its axis passes through the grasping point p_i (i.e. the value of the associated helicoidal vector field at the point p_i is zero: $T(p_i) = 0$).

3 Main Results

In this section we seek to explore a geometric entity in the configuration of the three fingered grasp which is then exploited in devising a method for finding the bases of the twist and wrench spaces. In other words, if $\mathcal{B} = \{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4, \mathbf{X}_5, \mathbf{X}_6\}$ is a basis of $\mathcal{D}$, then the expansion of $\mathbf{X} \in \mathcal{D}$ with respect to $\mathcal{B}$ can then be written as:

$$\mathbf{X} = \theta_1\mathbf{X}_1 + \theta_2\mathbf{X}_2 + \theta_3\mathbf{X}_3 + \theta_4\mathbf{X}_4 + \theta_5\mathbf{X}_5 + \theta_6\mathbf{X}_6$$

where $\theta_i \in \mathbf{R}$. In the three fingered grasp, the basis of $\mathcal{D}$ can be identified by defining the vertices of a tetrahedron where vertices consist of the grasping points and to be defined auxiliary point p (we call this tetrahedron the *invariant configuration of the three fingered grasp* (Figure (3)). Let the vectors $\mathbf{u}_i$ be directed along the edges of this invariant configuration. Let $\mathbf{X}_i \in \mathcal{D}$ be defined as:

$$\omega_{\mathbf{X}_i} = \mathbf{u}_i$$

Then $(\mathbf{X}_1, \cdots, \mathbf{X}_6)$ is a basis of the $\mathcal{D}$.

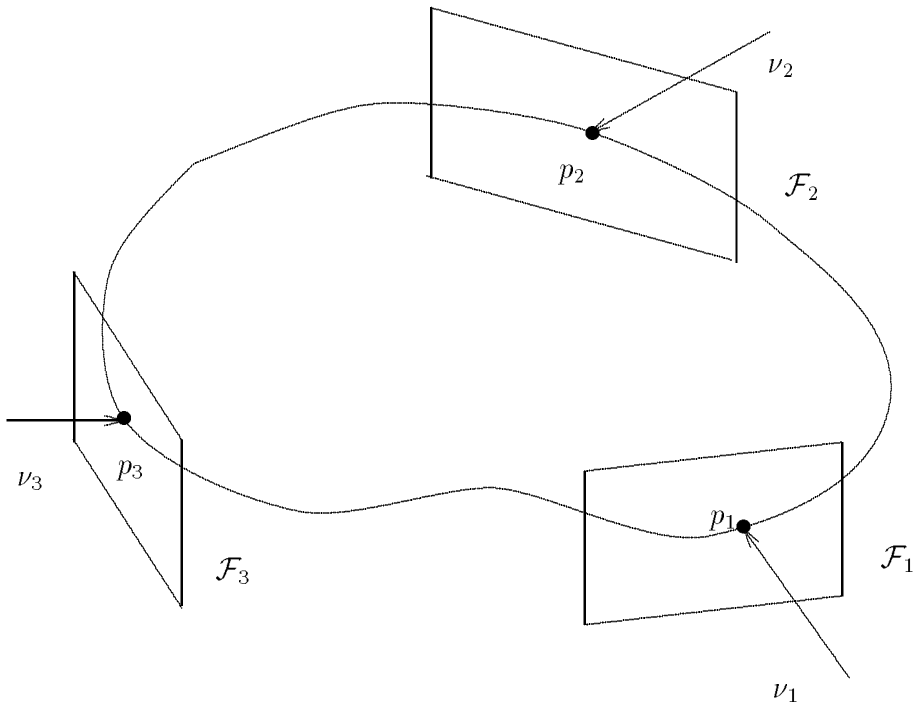

Figure 2: Definition of a three-fingered grasp

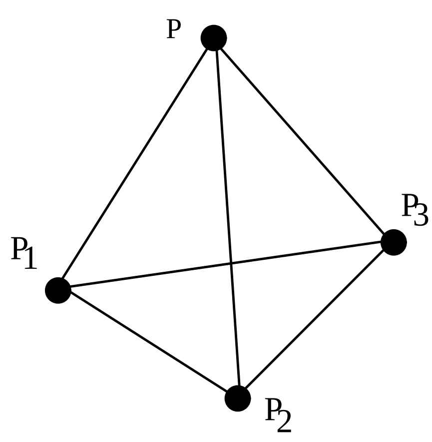

Figure 3: Definition of the invariant configuration in the three fingered grasp

In the following, the objective is to define such invariant configuration of the grasp where a wrench $\mathbf{W}$ and a twist $\mathbf{T}$ can be expanded to their corresponding values at the grasping points. Let the normalized screw $\mathcal{S}_i$ $(i = 1, 2, 3)$ is defined such that:(Figure (4))

$$\begin{cases} \mathcal{S}_i = \Pi_j \cap \Pi_k \text{ , if } \Pi_j \text{ and } \Pi_k \text{ are not parallel} \\ \mathcal{S}_i = \Omega\nu_j \text{ , if } \Pi_j \text{ and } \Pi_k \text{ are parallel} \end{cases}$$

When Π_j and Π_k are parallel, $\Omega\nu_j = \pm\Omega\nu_k$ and we can also have $\mathcal{S}_i = \Omega\nu_k$.

The normalized screw Σ_i $(i = 1, 2, 3)$ is also defined such that:(Figure (5))

$$\Sigma_i(p_j) = \Sigma_i(p_k) = 0.$$

i.e. the axis of Σ_i is the line $p_j p_k$.

The screws Σ_1, Σ_2 and Σ_3 generate a three-system of screws $\mathcal{T}$, that is the set of screws:

$$\alpha_1\Sigma_1 + \alpha_2\Sigma_2 + \alpha_3\Sigma_3 \text{ with } \alpha_1, \alpha_2, \alpha_3 \in \mathbf{R} \tag{8}$$

which is also the set of screws with zero pitch and with axis lying in the plane p_1, p_2, p_3.

Let u_i be the normalized vector along the line pp_i where point p is the intersection of all the tangent planes Π_i (Figure (6)). Whenever p is at infinity in the case where ν_1, ν_2 and ν_3 are parallel to the same plane or when two tangent planes are parallel, we may choose $u_1 = u_2 = u_3 = u$.

Let us also define ξ_i be the such that:

$$\xi_i \in \mathcal{Z}_i, \quad \omega_{\xi_i} = u_i, \quad i = 1, 2, 3,$$

then $\xi_i \in \mathcal{F}_i, \quad i = 1, 2, 3.$

For all i, it is possible to choose a screw η_i such that:

$$\eta_i \in \mathcal{F}_i \cap \mathcal{T}, \text{ and } \eta_i \neq 0. \tag{9}$$

A screw η_i like it was defined in equation (9) is in $\mathcal{Z}_i$ if $\alpha_i = 0$, that is if:

$$\eta_i = \alpha_j\Sigma_j + \alpha_k\Sigma_k, \qquad \alpha_j, \alpha_k \in \mathbf{R}$$

In order for this screw to be also in $\mathcal{F}_i$ it is necessary that,

$$[\Omega\nu_i \mid \eta] = [\Omega\nu_i \mid \alpha_j\Sigma_j + \alpha_k\Sigma_k] \tag{10}$$

$$= \alpha_j [\Omega\nu_i \mid \Sigma_j] + \alpha_k [\Omega\nu_i \mid \Sigma_k] = 0 \tag{11}$$

Solving for α_j and α_k, we have:

$$\alpha_j = [\Omega\nu_i \mid \Sigma_k], \qquad \alpha_k = -[\Omega\nu_i \mid \Sigma_j],$$

and thus

$$\eta_i = [\Omega\nu_i \mid \Sigma_k] \Sigma_j - [\Omega\nu_i \mid \Sigma_j] \Sigma_k. \tag{12}$$

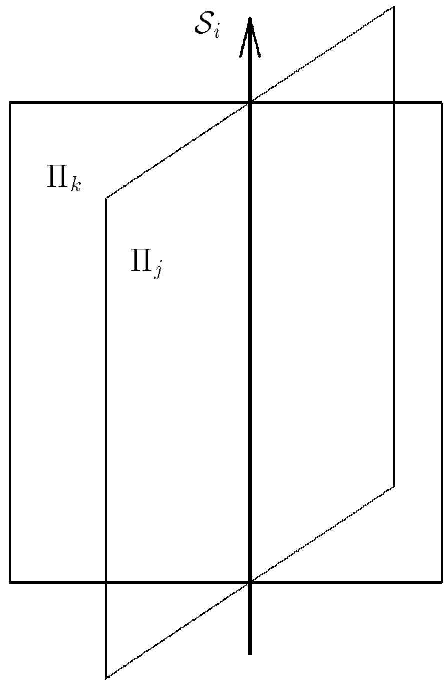

Figure 4: Definition of screw $\mathcal{S}_i$

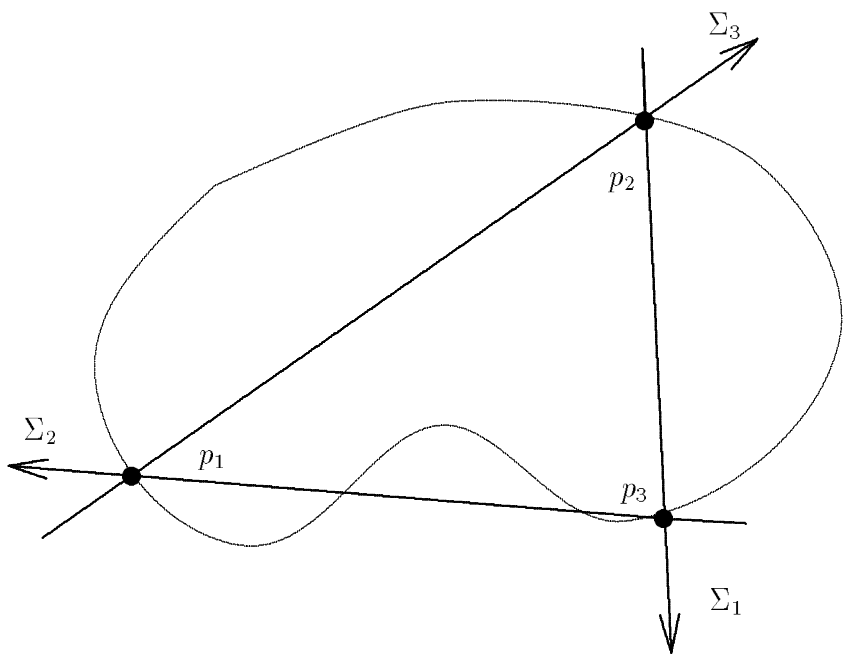

Figure 5: Definition of screw Σ_i

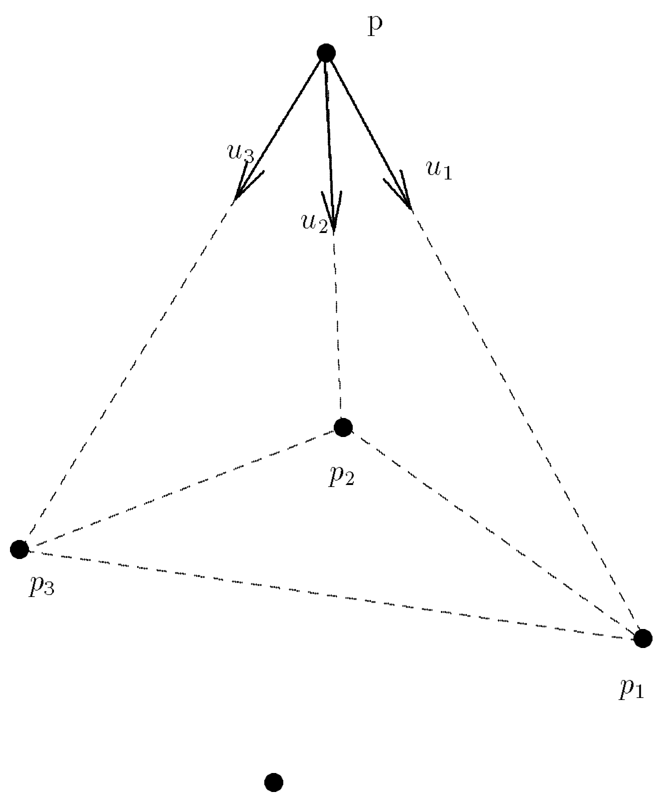

Figure 6: Definition of vectors u_i

For each i; ξ_i and η_i are linearly independent screws and therefore, $\{\xi_i, \eta_i\}$ is a basis of $\mathcal{F}_i$. Let $\mathbf{W}$ be a wrench acting on the grasped object. Then this wrench can be expanded into the friction forces at the contact point as:

$$\mathbf{W} = \sum_{k=1}^{3} x_k \xi_k + y_k \eta_k, \qquad x_k, y_k \in \mathbf{R}, \tag{13}$$

then,

$$\begin{aligned} [\mathbf{W} \mid \mathcal{S}_i] &= y_i \, [\eta_i \mid \mathcal{S}_i] \\ [\mathbf{W} \mid \Sigma_i] &= x_i \, [\xi_i \mid \Sigma_i] \end{aligned}$$

If (p_1, p_2, p_3, p) are not in the same plane implies that $[\xi_i \mid \Sigma_i] \neq 0$ and $[\eta_i \mid \mathcal{S}_i] \neq 0$ and the solution is: $x_i = \frac{1}{[\xi_i|\Sigma_i]} [\mathbf{W} \mid \Sigma_i]$ and $y_i = \frac{1}{[\eta_i|\mathcal{S}_i]} [\mathbf{W} \mid \mathcal{S}_i]$.

Similarly, one can exploit the geometry of an invariant configuration of the three fingered grasp for determining the instantaneous motions of the finger-tips as a function of the desired motion of the grasped object.

Here let point $p \in \varepsilon$, and $\mathcal{Z}_p$ be the vector subspace of $\mathcal{D}$ defined by:

$$\mathcal{Z}_p = \{\mathbf{X} \mid \mathbf{X}(p) = 0\} \tag{14}$$

and also, let us define screws Ψ_1 , Ψ_2 , $\Psi_3 \in \mathcal{D}$ such that:

$$\Psi_i(p_i) = \Psi_i(p) = 0 \tag{15}$$

Let us think of the point p as the center of a spherical joint (the wrist) and the points p_1, p_2 and p_3 as particles belonging to the grasped object (and which are moved by the finger-tips). The screw systems $\mathcal{Z}_p$ and $\mathcal{T}$ (equation (8)) describe simple motions relative to the conditions of the grasp.

A twist of the object which is described by a member of $\mathcal{Z}_p$ is a rotation about the point p and can be produced by a motion of the wrist alone. In particular, Ψ_i describes a rotation about the line pp_i: the contact point p_i remains fixed whereas p_j and p_k move.

A twist of the object which is described by a member of $\mathcal{T}$ is a rotation about an axis lying in the plane $p_1p_2p_3$ and can be produced by the motion of the fingers. In particular Σ_i describes a rotation about the line p_jp_k: the contact point p_i is moved whereas the contact points p_j and p_k remain fixed. In other words, points p, p_1, p_2, p_3 are not in a same plane, then:

$$\mathcal{D} = \mathcal{T} \oplus \mathcal{Z}_p \tag{16}$$

Moreover $\{\Sigma_1, \Sigma_2, \Sigma_3, \Psi_1, \Psi_2, \Psi_3\}$ is a basis of $\mathcal{D}$ such that $\Sigma_1, \Sigma_2, \Sigma_3 \in \mathcal{T}$ and $\Psi_1, \Psi_2, \Psi_3 \in \mathcal{Z}_p$.

Let $\mathbf{T} \in \mathcal{D}$ be a twist of the grasped object. Then this twist can be expanded as:

$$\mathbf{T} = \sum_{i=1}^{3} x_i \Sigma_i + \sum_{i=1}^{3} y_i \Psi_i \qquad x_i, y_i \in \mathbf{R} \tag{17}$$

where x_i and y_i are the magnitudes of the screws.

The corresponding amplitude of the twist about the defined basis can be obtained by forming the inner products of Σ_i and Ψ_i with both side of equation (17) respectively we can obtain:

$$[\mathbf{T} \mid \Sigma_i] = y_i [\Sigma_i \mid \Psi_i] \qquad [\mathbf{T} \mid \Psi_i] = x_i [\Sigma_i \mid \Psi_i]$$

Assuming that points p_1, p_2, p_3 and p are not on the same plane (which in general the case) implies that $[\Sigma_i \mid \Psi_i] \neq 0$ and we can solve for the amplitudes of the twists as:

$$x_i = \frac{[\mathbf{T} \mid \Psi_i]}{[\Sigma_i \mid \Psi_i]}, \qquad y_i = \frac{[\mathbf{T} \mid \Sigma_i]}{[\Sigma_i \mid \Psi_i]} \tag{18}$$

Hence, the desired twist of the grasped object can be obtained as a linear combination of Σ_i and Ψ_i.

4 Example

A cylindrical object of radius 1 is grasped by three fingers. The contact points are given as: $p_1 = (1, 0, 0), p_2 = (-0.5, 0.866, 0.)$ and $p_3 = (-0.5, -0.866, 0.)$. The normalized screws representing the contact normals (n_1, n_2, n_3) are written as:

$$\begin{aligned} \nu_1 &= (-1, 0, 0; 0, 0, 0), \\ \nu_2 &= (0.5, -0.866, 0.0; 0, 0, 0), \\ \nu_3 &= (0.5, 0.866, 0.0; 0, 0, 0). \end{aligned}$$

The objectives is to determine the required friction forces between the finger-tips and the grasped object and to determine the twist of each finger which can result in the desired motion of the grasped object defined by:

a) $\mathbf{T} = \Sigma_1$;

b) $\mathbf{T} = (0, 0, 1; 0, 0, 0)$, (i.e.rotation about the $z - axis$) and

c) $\mathbf{T} = (0, 0, 0; 0, 0, 1)$, (i.e. translation along the $z - axis$).

From the definition, for the case when ν_1, ν_2 and ν_3 are parallel to the same plane, we choose $u_1 = u_2 = u_3 = u = (0, 0, 1)$. As a result the normalized screws ξ_1, ξ_2 and ξ_3 can be computed to be:

$$\begin{aligned} \xi_1 &= (0, 0, 1; 0, -1, 0), \\ \xi_2 &= (0, 0, 1; 0.866, 0.5, 0), \\ \xi_3 &= (0, 0, 1; -0.866, 0.5, 0). \end{aligned}$$

From definition, the screws Σ_1, Σ_2 and Σ_3 which are normalized screws through points (p_2, p_3), (p_3, p_1) and (p_1, p_2) respectively are determined using the following general *Plücker* line coordinates representation, i.e. for two point $p_i = (x_i, y_i, z_i)$ and $p_j = (x_j, y_j, z_j)$:

$$\begin{aligned} &L = x_j - x_i, \\ &M = y_j - y_i, \\ &N = z_j - z_i, \\ &P = y_i N - z_i M, \\ &Q = z_i L - x_i N, \\ &R = x_i M - y_i L, \end{aligned} \tag{19}$$

the normalized screw Σ_1, Σ_2 and Σ_3 can be obtained as: (screws with zero pitch)

$$\begin{aligned} \Sigma_1 &= (0, -1, 0; 0, 0, 0.5), \\ \Sigma_2 &= (0.866, 0.5, 0; 0, 0, 0.5), \\ \Sigma_3 &= (-0.866, 0.5, 0; 0, 0, 0.5), \end{aligned} \tag{20}$$

similarly, $\mathcal{S}_i$ can be computed to be:

$$\begin{aligned} \mathcal{S}_1 &= (0, 0, 1; 0, 2, 0), \\ \mathcal{S}_2 &= (0, 0, 1; -1.732, -1, 0), \\ \mathcal{S}_3 &= (0, 0, 1; 1.732, -1, 0). \end{aligned}$$

From equation (12), we have:

$$\begin{aligned} \eta_1 &= (0, 0.866, 0; 0, 0, -0.433), \\ \eta_2 &= (-0.749, -0.433, 0; 0, 0, -0.749), \\ \eta_3 &= (0.749, -0.433, 0; 0, 0, -0.433). \end{aligned}$$

Let the only force which can act on the grasped object be the object's own weight of 10(lbf) acting at its center of gravity located at point (o) or, $\mathbf{W} = (0, 0, -10, 0, 0, 0)$. Then, we can compute $[\mathbf{W} \mid \Sigma_i] = -5$ for $(i = 1, 2, 3)$. Also, for the grasp configuration of this example we have: $[\mathbf{W} \mid \mathcal{S}_i] = 0$ for $(i = 1, 2, 3)$. The product $[\xi_i \mid \Sigma_i] = 1.5$ for $i = 1, 2, 3$.

Therefore, the friction forces associated with the three-fingered grasp configuration are computed to be:

$$\begin{aligned} F_1 &= x_1\xi_1 &= (0, 0, -3.33; 0, 3.33, 0), \\ F_2 &= x_2\xi_2 &= (0, 0, -3.33; -2.883, -1.665, 0), \\ F_3 &= x_3\xi_3 &= (0, 0, -3.33; 2.883, -1.665, 0). \end{aligned}$$

Now for the case of manipulation let coordinates of point p be defined as $(0, 0, 1)$. Based on the definition of Ψ_i (equation (15), we can also obtain:

$$\begin{aligned} \Psi_1 &= (1, 0, -1; 0, 1, 0), \\ \Psi_2 &= (-0.353, 0.612, -0.707; -0.612, -0.353, 0), \\ \Psi_3 &= (-0.353, -0.612, -0.707; 0.612, -0.353, 0). \end{aligned} \tag{21}$$

From these screw descriptions representing the geometry of the grasp, we have the following magnitudes for the inner products defined in equation (17):

$$\begin{aligned} &[\Sigma_1 \mid \Psi_1] = -1.5, \quad [\Sigma_1 \mid \Psi_2] = 0, \quad [\Sigma_1 \mid \Psi_3] = 0 \\ &[\Sigma_2 \mid \Psi_1] = 0, \quad [\Sigma_2 \mid \Psi_2] = -1.058, \quad [\Sigma_2 \mid \Psi_3] = 0 \\ &[\Sigma_3 \mid \Psi_1] = 0, \quad [\Sigma_3 \mid \Psi_2] = 0, \quad [\Sigma_3 \mid \Psi_3] = -1.058. \end{aligned} \tag{22}$$

For case a), we have the following decomposition of the desired twist $\mathbf{T}$ into its components (i.e. the trivial case). For this case, $[\mathbf{T} \mid \Sigma_i] = 0$ and $[\mathbf{T} \mid \Psi_i] = 0$ for $i = 2, 3$ and $[\mathbf{T} \mid \Psi_1] = -1.5$. As a results, we have:

$$\mathbf{T} = (0, -1, 0; 0, 0, 0.5)$$

In this example, since the axis of twist is Σ_1, the contact point p_1 has instantaneous rotation in a circular path about the axis $p_2 p_3$.

For the case b), the desired twist of the object is defined as the rotation about the $z-axis$ or $\mathbf{T} = (0,0,1;0,0,0)$. Here, the nonzero inner products are calculated to be:

$$[\mathbf{T} \mid \Sigma_1] = [\mathbf{T} \mid \Sigma_2] = [\mathbf{T} \mid \Sigma_3] = 0.5.$$

Following equation (17), the desired twist of the object can be obtained by the sum of the twist of each finger-tips, or

$$\mathbf{T} = y_1 \Psi_1 + y_2 \Psi_2 + y_3 \Psi_3,$$

where $y_1 = -0.333$, $y_2 = -0.472$ and $y_3 = 0.472$ are the amplitudes of the twists about Ψ_1, Ψ_2 and Ψ_3.

The desired twist of the object $\mathbf{T}$ can be accomplished by sequentially holding finger i fixed while moving the fingers j and k such that the final desired twist of the object is accomplished. Another alternative would be to rotate the wrist without moving the fingers.

For the case c) where the desired twist of the object is given as translation along the $z-axis$ or : $\mathbf{T} = (0,0,0;0,0,1)$. For this case, $[\mathbf{T} \mid \Sigma_i] = 0$ for $i = 1,2,3$ and we have:

$$[\mathbf{T} \mid \Psi_1] = [\mathbf{T} \mid \Psi_2] = [\mathbf{T} \mid \Psi_3] = 0.668$$

As a result, the desired twist of the object can be obtained by sum of the twists about Σ_1, Σ_2 and Σ_3.

5 Conclusions and Future work

When grasping an object with dexterous mechanical hand, the contact points can be defined in such way that the grasp can be in force closure. The other important consideration in such interaction is the magnitudes of grasping forces that the fingers can exert on the object so the slippage of the object can be avoided. There are two approaches to determine the grasping forces, one is to select an upper bound on the grasping forces such that the slippage of the object is avoided for a given range of the external forces. The other is to compute the expected magnitudes as a function of the task forces and adjust the magnitudes of the grasping forces on-line.

This paper presented an elegant method for computing the friction forces (i.e. grasping forces) of the three fingered grasp as a function of the external force. In addition, a method is presented to determine the magnitude of the finger-tips motions as a function of the desired object motion. The method exploits the geometrical entity which is formed by the contact points and an auxiliary point. The method uses screw geometry and inner product spaces.

The framework of this paper can be extended to include dynamic wrenches and the inertial properties of the grasped object. Example of this can be the case when the object is picked by three fingered end-effector and while being held between the fingers, the arm moves along a certain trajectory. In this way, the results can be used in the real-time dynamics manipulation of the object. This topic is currently under investigation.

Acknowledgments

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