EXTENSIONS OF NONLINEAR CONTROL CONCEPT TO OBJECT MANIPULATION WITH FINGER RELOCATION

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Abstract: The paper provides a powerful basic concept for object manipulation with finger relocation and extends the previous approaches in the area of stratified manipulation. The algorithm gives also some improvements of the developed software for smooth and stratified motion planning (MP). The key element of the approach is based strongly on the theory of stratified control for nonlinear systems. The discussed manipulation concept exploits the information about the shape of the object and endeavors to discover its most important physical properties. In connection with the subject, the paper investigates the finger gaiting manipulation (based on finger relocation) in the context of stratified nonlinear control.

Keywords: Nonlinear control, Manipulation, Differential geometric methods, Differential equations, Robotic manipulators

1. INTRODUCTION

In the common sense, the intention of a manipulation task is to find a motion trajectory of the agents that move the object from a given initial grasp to the desired configuration. As a requirement, the control has to insure collision free paths for all agents and suitable forces for stable grasp and manipulation. Several possible approaches of the essential mechanics of manipulated objects are examined in (Salisbury and Mason, 1985). Some interesting approaches can be found in (Vass *et al.*, 1999). Existing nonlinear control methods prefer continuous system of motion equations which is usually not satisfied for numerous manipulating tasks. One of the most important situations is when different constraints affect cyclically the system. For example, legged robotic systems and multifingered dextrous hands with alternating contacts belong to this subset.

In the case of multifingered system, each possible finger configuration is represented with one of the strata in configuration space and specified with the fingers and the surface of the object which are in contact. If a finger makes or brakes contact then the equations of motion change discontinuously and the system moves to another stratum. In fact, a smooth nonlinear system can be regarded as a special stratified system where the only stratum is the whole configuration space (without constraints). Therefore, smooth motion planning (SMP) algorithm is a good start point in the understanding of the problem. The Appendix summarizes the most important results of the SMP algorithm originated from (Lafferriere and Sussmann, 1991), and (Sussmann, 1992).

The paper is organized as follows. Section 2 discusses the theoretical background of stratified control and stratified MP (Goodwine, 1998*a*) and (Goodwine, 1998*b*). In fact, this method relies strongly on the SMP algorithm. However, the stratified MP method works on some special system, namely on the *Bottom Stratified System* and the *Extended Bottom Stratified System.* The solution of MP problem is determined by different kind of sequences. More precisely, the solution contains a sequence of flows represented by vector fields, a sequence of time indexes described along the trajectory and a sequence of control signals. Flows determine the trajectory and the control is constant between two time indexes. In the practice, the above mentioned procedures may easily lead to some questions and limitations caused by nonsmooth object surfaces, the critical distance of a trajectory segment, and time scaling. Some improvements for the task independent part of stratified control algorithm are proposed in Section 3. The object with edges plays a distinguished role in a manipulation problem. Section 4 is devoted to this problem and gives some ideas to overcome the difficulties in the frame of stratified MP.

In order to execute a successful manipulation, some properties (for example the shape) of the object should be known. In the focused problem, the manipulation cannot be independent of the geometry of the object. The purpose of the paper is to offer an extended concept using (stratified) nonlinear control methods for practical manipulation problems. Future goal of the research is to implement the concept and develop a software package which makes possible to obtain the most important physical attributes (mass, center of mass) of the object during the manipulation.

2. THEORETICAL BACKGROUND OF STRATIFIED CONTROL

This section is a brief overview of stratified control approach which was elaborated by Goodwine (Goodwine, 1998*a*) and (Goodwine, 1998*b*). Consider a configuration space specified by the states of the system. In many cases, the system cannot be determined by one smooth manifold because the equations of motion depend on the actual state vector. The different types of constraints decompose the configuration space into in itself smooth manifolds (strata).

Definition 1. A set $\aleph \subset \mathbb{R}^n$ defined by union of smooth manifolds (i.e. strata) is said to be *regularly stratified set.*

Definition 2. The system is *stratified* if its configuration space is defined by regularly stratified sets.

In each stratum a smooth nonlinear system is defined by vector fields. The main problem occurs when one wants to move the system from one stratum to another one. For the sake of convenience, it is helpful to introduce some further notations. Consider an object manipulation problem with "two fingered" hand. Let $M \equiv S_0$ be the whole configuration space where no finger is in contact with the object. Let the stratum $S_i \subset M$ be a codimension one submanifold where only one finger is in contact with the object. Roughly speaking, this stratum corresponds to dimension $n-1$ manifold in the configuration space. Let $S_{ij} = S_i \cap S_j$ where both the ith finger and jth finger are in contact with the object. Recursively, $S_I = S_{i_1 i_2 \ldots i_k} = S_{i_1} \cap S_{i_2} \cap \cdots S_{i_k}$ where $I = i_1 i_2 \ldots i_k$ is a multi-index (Isidori, 1996). The stratum with lowest dimension which includes the point x is said to be the *bottom stratum.* A stratum is called *lower stratum* if its dimension is lower than the dimension of the other one. The *higher stratum* is defined vice versa.

Theorem 1. (Goodwine) Let $T_{x_0}M$ be the tangent space of M in x_0 and let $\bar{\Delta}_{S_j} |_{x_0}$ denote the involutive closures of a distribution which is spanned by the vector fields of a stratum S_j in x_0. If there exists a nested sequence of strata $x_0 \in S_p \subset S_{p-1} \subset \cdots \subset S_1 \subset S_0$, such that the involutive closures of distributions (of strata) fulfill $\sum_{j=0}^{p} \bar{\Delta}_{S_j} |_{x_0} = T_{x_0}M$ then the system is locally stratified controllable from x_0.

The control methods elaborated for smooth systems show an important difficulty in the general MP problem because different strata are described by different equations of motion and there are strata in which the system is not controllable. The idea of stratified control is to define a common space where all the vector fields can be considered. It will be the bottom stratum. The following example makes a better understanding. Consider a manipulation system with two fingers whose configuration space is showed by Fig. 1. It is typical that the system is not controllable on S_{12}. The manipulation process can be described by a sequence of flows where vector field $g_{1,1}$ moves the system off of S_{12} onto S_1 ("finger 1" disconnects the object), vector field $g_{2,1}$ moves the system off of S_{12} onto S_2 ("finger 2" disconnects the object), $g_{1,2}$ is defined on stratum S_1, $g_{2,2}$ is defined on stratum S_2. It is beneficial to distinguish two sets of vector fields.

Definition 3. A vector field is said to be a *moving off* vector field if the existence of contact between the finger and the object depend on it. In other words, the moving off vector fields switches be-

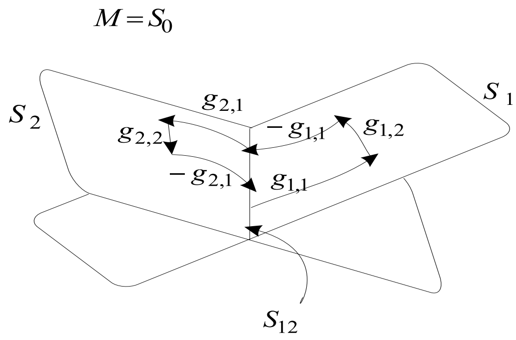

Fig. 1. *Flow sequences in stratified configuration space.*

tween strata.

Definition 4. A vector field is said to be a *moving on* vector field if it does not leave the actual stratum.

If the moving on vector fields commute with moving off vector fields (i.e. by definition, their Lie brackets are zero) then the flow sequence can be rearranged and reduced to bottom stratum. Additionally, if the moving on vector fields in the higher strata are tangent to the bottom stratum S_{12} then the original and the reduced sequences achieve the same motion in the configuration space (Fig. 1).

Remark 1. If a vector field on higher stratum (for example $g_{1,2}$) is not tangent to "its" substratum then it is possible to modify this vector field such that the tangency requirement is satisfied.

Corollary 1. If all the vector fields which separate a finger from the object are decoupled from all vector fields defined on the substratum and higher strata (in other words, their Lie brackets are zero) then the moving between higher and lower strata is possible.

The realization become easier if the following stronger assumption is satisfied.

Assumption 1. Assume that the tangent vector $\{\frac{\partial}{\partial h_i}\}$ can be produced as a linear combination of control inputs where h_i is the distance between the ith finger and the object. Additionally, assume that the effective equations of motion in a stratum are not influenced by the distance of the "contact free" finger.

The assumption guarantees that the Lie brackets between moving off and moving on vector fields are zero. Furthermore, the tangency requirements will also be automatically satisfied. The assumption for kinematic systems is usually satisfied, however, this is not necessarily true for dynamic systems.

Remark 2. The paper deals with kinematic model. It is clear from the earlier discussion that the stratified motion planning algorithm needs an extended system on the bottom stratum (as a common space). In order to obtain such a system, one has to take the union set of vector fields in all strata and collect them into one fictitious system as a *bottom stratified system.* If the above assumption is fulfilled then the bottom stratified system will contain all the vector fields from every stratum omitting the vector fields that move the fingers off of the object. In the opposite case, it consists of the all vector fields that commute with the moving off vector fields (i.e. the vector fields which disconnect a finger from the object). The *bottom stratified extended system* consists of the Lie brackets among all vector fields of bottom stratified system.

Example 1. (Multiple Stratified System)

$$\begin{aligned} S_0 &: \dot{x} = g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u_{0,n_0} \\ S_1 &: \dot{x} = g_{1,1}u^{1,1} + \cdots + g_{1,n_1}u_{1,n_1} \\ &\vdots \\ S_I &: \dot{x} = g_{I,1}u^{I,1} + \cdots + g_{I,n_I}u_{I,n_I} \end{aligned} \quad (1)$$

Example 2. (Bottom Stratified Extended System) $\dot{x} = g_{0,1}u^{0,1} + \cdots + g_{0,n_0}u_{0,n_0} + g_{1,1} |_{S_0} u^{1,1} + \cdots + g_{1,n_1} |_{S_0} u_{1,n_1} + \cdots + g_{I,1} |_{S_0} u^{I,1} + \cdots + g_{I,n_I} |_{S_0} u_{I,n_I} +$ *Lie brackets* where the notation $|_{S_0}$ refers to the vector fields which take a part in bottom stratified system, however, they are defined originally not in this stratum.

The algorithm of MP on this system solves also indirectly the stratified MP problem. The total stratified solution needs to insert suitable moving off vector fields in the flow sequence. However, it is already an elementary task.

3. IMPROVEMENTS OF STRATIFIED MOTION PLANNING ALGORITHM

The software package was developed based on MATLAB and Symbolic Toolbox for solving MP problems. During its applications for typical practical problems some improvements were necessary to increase the precision. The major problem of the stratified control and MP methods is to divide the trajectory into appropriate subsegments. The choice of the length of a subsegment may be critical in the point of view of the accuracy and the real time realization. The two aspects need compromise. Small subsegments risks the real time realization. However, large length of subsegments may cause accumulated tracking error and divergence. There exists a critical distance between the end points (Lafferriere and Sussmann, 1991) which assures the convergence, however its estimation arises to a hard question without theoretical answer. The proposed simple algorithm of step length modification starts with an initial distance D_1 between the start point p_1 and the end point q_1.If $\| q_i - p_i \| > D_{i-1}$ (where $p_i = q_{i-1}$) then let $D_i = D_{i-1}/2$ and insert some extra points between q_i and p_i through a line segment. If $\| q_i -$

$p_i \parallel < D_{i-1}/2$ then restore $D_i = 2D_{i-1}$. In order to illustrate a typical stratified MP algorithm, Fig. 2 shows the control characteristic on a simple (not manipulation) application. The figure illustrates a planned trajectory in a relevant subspace of the state space. The proposed method does not guarantee the convergence of MP in a given step but it avoids the convergence problem for the whole trajectory. The inserted additional points prevent the error accumulation and keep D_i in the near of critical distance D_r for long time. The earlier MP methods return with definite time for the trajectory (computed time). The concept of the next section and the developed software frame of stratified MP employ also a time-scaling which enables to prescribe arbitrary time point to each configuration point. The idea is to apply a factor for each input in the actual subsegment which is obtained with the quotient of the desired and computed time for the actual subsegment.

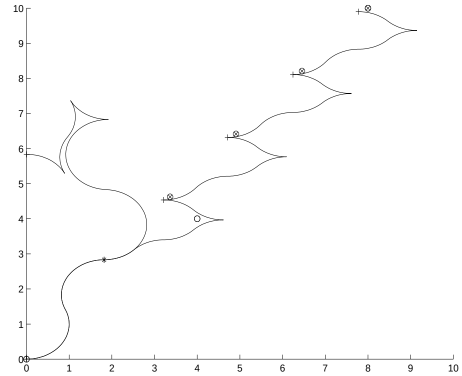

Fig. 2. *Stratified MP in a configuration space without (left branch) and with (right branch) improvement. The "o" denotes the prescribed points, the "+" denotes the reached points. The "x" denotes the additional inserted points by the algorithm.*

4. EXTENDED CONCEPTUAL MODELLING FOR FINGER GAITING MANIPULATION

The (smooth and stratified) MP for manipulation have a number of limitations. The earlier methods (Goodwine, 1998*a*), (Goodwine, 1998*b*) deal with object of special geometry. This section proposes a concept for a more general class of the objects having not exclusively smooth surfaces. The key idea is to decompose the surface of object into smooth submanifolds in the configuration space. Consider a convex object with two (smooth) surfaces (i.e. with edges) as an example. Let the kinematic model of the dextrous hand be given. Denote the subscripts of strata S the fingers which are in contact with the object and related to this, indicate the superscripts of S the surfaces which are in contact with the corresponding fingers.

Example 3. S_{12}^{11} denotes the stratum where both "finger 1" and "finger 2" are in contact with a "surface 1". Based on this convention, Fig. 3 illustrates the problem of stair climbing with fingers. The figure shows the strata and the flows but not the real stair. Dotted lines illustrate the heights which should be overstepped by the lifted fingers. In the beginning, both the fingers contact the "surface 1" (i.e. the ground) then the "finger 2" moves from the ground to the stair (i.e. to "surface 2") and after this, the "finger 1" moves also from the ground to stair. One cannot apply a pure stratified control method for the illustrated example because the union of bottom strata S_{12}^{11} and S_{12}^{22} on which the bottom stratified extended system is defined, is not smooth.

It is useful to define two foliations on the total configuration space. The foliation of "palm" variables (P_0) is associated to the position and orientation of palm frame and the foliation of manipulation variables (S_0) is associated to the internal shape variables and to the position and orientation of object in the palm frame. In the concept, the foliation of manipulation plays the most important role while the palm frame remains in calm. Indeed, the configuration subspace of manipulation variables is in itself a stratified configuration space (with strata S_0, S_1, S_2, S_{12} etc. where constraints are defined with the finger contacts). The idea is to separate the two configuration subspaces in point of view of manipulation problem. The proposed concept will be demonstrated in Fig. 3 for two fingers (agents) climbing a stair which can be considered as the simplest modelling of finger relocation on a nonsmooth object.

Step 1.) Move the object to (advantageous) manipulation position and orientation through the change of coordinates of palm frame while all fingers contact points are saved. Meanwhile, one can measure the finger-tip contact forces and try to obtain information and properties of the object. In this case, we perform a conventional (smooth) MP in the configuration subspaces associated to palm variables. The trajectory can be determined by forward P. Hall coordinates in the form $S = e^{\tilde{h}_1 B_1} \dots e^{\tilde{h}_{s-1} B_{s-1}} e^{\tilde{h}_s B_s}$ of formal differential equation (6) with the initial condition $S(0) = 1$. The algorithm assumes that the force closure stability is hold along the trajectory (Goodwine, 1998*b*).

Step 2.) Perform an agent relocation. The agent relocation is based on *stratified* MP in the bottom stratum of configuration space S_0 of manipulation variables. (In the example, the bottom strata S_{12}^{11},S_{12}^{12}, S_{12}^{21} and S_{12}^{22} play important role in this respect.) Indeed, this is a finger gaiting where the stages before and after a disconnection between ith finger and object (while all the $m-1$ other fingers remain in contact) corresponds the constraints

$$d\Phi_j\{g_{j,1}(x)u^{j,1} + \cdots + g_{j,n_j}(x)u^{j,n_j}\} = 0$$
$$j = 1 \ldots m, \quad (2)$$
$$d\Phi_j\{g_{j,1}(x)u^{j,1} + \cdots + g_{j,n_j}(x)u^{j,n_j}\} = 0$$
$$j = 1 \ldots m, j \neq i, \quad (3)$$

respectively. Φ_j are the level functions of strata S_j (for example the height of the fingers in the stratum). First time, let the fingers be relocated only on own smooth surfaces i.e. the start and end points of each finger are in the same smooth surface (but this may be different for different fingers). All the interesting trajectories should be taken which go through all the selected important contact points while each finger remains on its own smooth surface. In fact, it is a "scanning" in the sense that the relevant (contact) points of the trajectories should play distinguished role in point of view of physical attributes of the object. If the scanning finished (i.e. all the relevant points was reached) then the algorithm carries on with next step else repeats this step taking following relevant contact points on the surfaces of the corresponding fingers for identification purposes.
Step 3.) Relocate a finger from the surface to a new smooth surface. In fact, the primarily goal is to find a path between two bottom strata whose union is not smooth. Say, place the "finger 2" from ground to stair in the example (Fig. 3). The aim of control is to steer the system through S_1^1 from S_{12}^{11} to S_{12}^{12}. Since the start and end points can be considered in same stratum (S_1^1), this is a smooth MPP where the system is defined by the vector fields of actual stratum. The actual stratum "contains" the two bottom strata S_{12}^{11} and S_{12}^{12} determined by the fingers and surfaces which are in contact. It is worth remarking that the complete controllability is usually not guaranteed, i.e. the controllability Lie algebra generated by vector fields $f = \{f_1, \ldots, f_m\}$ of hand does not satisfy the *Lie Algebra Rank Condition* (LARC) (Sussmann, 1992) which restricts the reachable set from starting point. The idea is a stratified approach where we consider this actual stratum S_1^1 as a "bottom stratum" of higher strata which contain it. In the sequel, this actual stratum is called *common stratum.* Since the proposal allows to apply additional vector fields, the method offers a more robust solution. Of course it may occur that the extended system on the common stratum is not controllable. Fortunately, one has to find only a path between the two smooth bottom strata and not exactly between two points in these strata. If the relocation meets difficulties, jump Step 5 else continue with next Step.
Step 4.) If we have not enough information for the identification of physical properties (e.g. center of mass) of the object then jump Step 1 else go to final Step 6.
Step 5.) The finger relocation can meet the following problems:

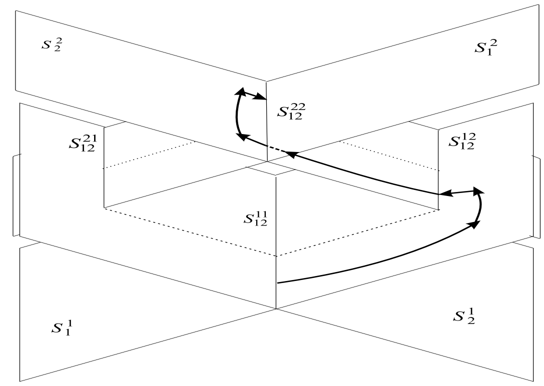

Fig. 3. *Configuration space with nonsmooth surface.*

- Object as obstacle. The obstacles indicate boundaries on the strata in configuration space (bounded by dot line in Fig. 3). If the computed trajectory meets with the obstacle then a smaller step size is needed.
- Another problem is the risk of finger collision during agent relocation. One can avoid the problem if the prescribed trajectory allows only separated workspace for the fingers. Depending on the strategy, the algorithm is carried on with Step 2 or 3.

Step 6.) End of algorithm.

5. CONCLUSIONS

The manipulation systems with discontinuous equations of motion need new control approaches like stratified control. The earlier stratified MP is able to deal with smooth objects but has also some limitations. A software package was developed for MP problems. The precision of the algorithms was improved by step length modification and the applicability was increased by time scaling. In order to overcome the more complex manipulation problem, an extended concept was developed for nonsmooth object manipulation with finger relocation of dextrous hands which gives also chance to identify the mechanical parameters of unknown objects in th initial phase. The software implementation of this concept is in progress.

ACKNOWLEDGEMENT

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6. REFERENCES

Goodwine, B. (1998*a*). Control of stratified systems with robotic applications. PhD thesis. California Institute of Technology.

Goodwine, B. (1998*b*). Stratified motion planning with application to robotic finger gaiting. In: *IFAC World Congress*. Beijing.

Isidori, A. (1996). *Nonlinear control systems*. Springer Verlag London Limited.

Lafferriere, G. and H.J. Sussmann (1991). Motion planning for controllable systems without drift. In: *Proceedings of the IEEE International Conference on Robotics and Automation*. Sacramento, CA, USA. pp. 1148–1153.

Salisbury, J. K. and M. T. Mason (1985). *Robot Hands and the Mechanics of Manipulation*. MIT Press.

Sussmann, H. J. (1992). New differential geometric methods in nonholonomic path finding. In: *Systems, Models, and Feedback: Theory and Applications* (A. Isidori and T. J. Tarn, Eds.). Birkhäuser. Boston. pp. 365–384.

Vass, G., S. Payandeh and B. Lantos (1999). Application of simulated annealing for object manipulation with multiple agents. In: *Proceedings of the Second IASTED International Conference, Control and Applications*. pp. 566–571.

7. APPENDIX

The solution of the SMP problem is a control in the configuration space that steers the starting point p to the end point q. The following definitions and results developed by Lafferiere and Sussmann (Lafferriere and Sussmann, 1991), (Sussmann, 1992) are fundamental for stratified control.

Definition: A *Nilpotent Lie algebra* with order k is defined by Lie algebra L where all the Lie brackets $[v_1, [v_2, \ldots, [v_k, v_{k+1}] \ldots]]$ equal to zero.

Definition: The system Σ is said to be *nilpotent* if its controllability Lie algebra $L(f)$ is a nilpotent Lie algebra.

Assumptions:

i) The control system has no drift, i.e.

$$\Sigma \quad : \dot{x} = u_1 f_1(x) + \ldots + u_m f_m(x). \quad (4)$$

ii) $f = \{f_1, \ldots, f_m\}$ are *real analytic* vector fields on $\mathbb{R}^n$.

iii) Σ is completely controllable.

Remark: A system has no drift if the right hand side of (4) has no term which is independent from u but depends on x. A system $\dot{x} = Ax + Bu$ has drift, while $\dot{x} = Bu$ has not. In order to apply the theory, fingers will be described by (kinematical) model without drift.

The strategy. Lafferriere and Sussmann propose to extend the system Σ to

$$\begin{aligned} \Sigma_e : \ \dot{x} &= v_1 f_1(x) + \ldots + v_m f_m(x) \qquad (5) \\ &+ v_{m+1} f_{m+1}(x) + \ldots + v_r f_r(x). \end{aligned}$$

where vector fields $f_{m+1}, \ldots, f_r$ are defined by higher order Lie brackets of f_i selected so that $span\{f_1(x), \ldots, f_r(x)\} = \mathbb{R}^n$ and $v_1, \ldots, v_r$ are fictitious controls. The strategy consists of two main steps:

1. Find a control v that steers the extended system (5) from p to q.
2. Compute control u for original system Σ that perfectly substitutes the fictitious control v.

The first step. One has to find only a curve as path between p and q. Since the vector fields of Σ_e span the whole configuration space, the simplest case for smooth system is the straight-line segment.

The second step. This step consists of substeps based on the properties of P. Hall basis. The P. Hall basis is totally ordered and a degree belongs to it. The P. Hall basis is a set of Lie brackets where the order of right element of any Lie bracket from the set is equal or greater than the order of left element. Especially, $\hat{L}(X_1, \ldots, X_m)$ denotes noncommutative formal power Lie series belonging to the Lie algebra $L(X_1, \ldots, X_m)$ where X_i denotes vector field. Moreover, $\hat{L}_k(X_1, \ldots, X_m)$ is the nilpotent version of $\hat{L}(X_1, \ldots, X_m)$ where k is the order of nilpotency. Short forms are L_k^m, $\hat{L}_k^m$ etc. A basis B' of Lie algebra $L(X_1, \ldots, X_m)$ can be immediately obtained from a Philip Hall basis of $L(X_1, \ldots, X_m)$. Furthermore, the set $\{M \in B' : degree(M) \le k\}$ represents a basis of L_k^m where L_k^m is a free nilpotent Lie algebra in $(X_1, \ldots, X_m)$ with order k.

First, one has to determine the Philip Hall basis and the order of system nilpotency. Then the formal differential equation

$$\begin{aligned} \Sigma_f e : \ \dot{S}(t) &= S(t)(v_1(t) f_1 + \ldots + v_m(t) f_m \\ &+ v_{m+1}(t) f_{m+1} + \ldots + v_r(t) f_r). \\ S(0) &= 1 \qquad (6) \end{aligned}$$

has to be solved in P. Hall coordinates on a special nilpotent Lie group. If $B_1, B_2, \ldots, B_m$ are P. Hall basis of the Lie algebra $L_k(X_1, \ldots, X_m)$ then any solution S can be uniquely expressed in the forms

$$\begin{aligned} S &= e^{h_s B_s} e^{h_{s-1} B_{s-1}} \cdots e^{h_1 B_1} \qquad (7) \\ S &= e^{\tilde{h}_1 B_1} \cdots e^{\tilde{h}_{s-1} B_{s-1}} e^{\tilde{h}_s B_s} \end{aligned}$$

where h_i is called *backward P. Hall coordinates* and $\tilde{h}_i$ is called *forward P. Hall coordinates*. Based on the formal solution S, the real solution $x(t) = \bar{x} e^{a_1(x) f_1} e^{a_2(x) f_2} \cdots e^{a_r(x) f_r}$ can be determined by symbolic computation of the P. Hall coordinates $a_1(t), \ldots, a_r(t)$.

Finally, the control u is determined from these P. Hall coordinates. The solution is a sequence of flows and constant control signals.

Remark: If the system is not nilpotent then the solution is only an approximation.