Planning Velocities of Free Sliding Objects for Dynamic Manipulation

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Abstract—
In this paper, a novel numerical approach is proposed to solve the initial velocities of the free sliding object for given initial and final configurations. To find the desired initial velocities for free sliding objects is a key step for implementing impulse manipulation. In order to plan the initial velocities, the motion of free sliding objects is modeled as a set of 6 first order differential equations, and the planning problem is formulated as a free boundary value problem(FBVP). Through a simple transformation, the FBVP is reduced to a standard Two-point boundary value(TPBV) problem. Quasi-Newton based optimization procedures are utilized to solve the planning problem. Unlike existing approaches, the proposed method does not require qualitative motion characteristics, thus it can be used for objects with general shape and arbitrary pressure distribution. The method always gives faster convergence rate than other methods based on characteristics analysis. Simulation results on polygonal objects with three to five vertices are used to demonstrate the planning method.

I. Introduction

In robot manipulation, one of the basic task is to move object from initial configuration to goal configuration. One possible method is to grasp objects rigidly and then move them. The other is to move objects by nonprehensive techniques such as pushing, throwing, batting and striking[7]. Dynamic manipulation includes hopping, juggling, tapping, and batting, and impulse planar manipulation. Impulse planar manipulation were studied in [6][5][4]. The problem was decomposed into the impact problem and the inverse sliding problem. The inverse sliding problem is to determine the initial velocities required for the object to slide to the desired displacement(translation and rotation) based on the dynamics of sliding object. The dynamics of sliding motion of disks and rings has been studied in [9], some properties of the motion were proposed. In [3], concept of limit surface was introduced to studied the relation between the motion of the slider and the frictional force. Owing to the complex dynamics of the free sliding object, for a given displacement of the object, it was founded that it is impossible to determine the desired velocities analytically.

The inverse sliding problem for the class of axisymmetric objects is addressed in [5]. Axisymmetric objects are those which have a pressure distribution that is a function of radius of the object only, and have the property that they always slide in a straight line. The planning is to find the initial linear velocity along the line, and the associated rotational velocity. Using the properties of monotonicity of displacement with respect to the initial velocities, a numerical approach is developed to find the desired initial velocities through subdividing the initial velocity space. However, the monotonicity properties do not hold for nonaxismmetric objects, which limits the applications of the impulse manipulation method. Recently the impulse manipulation has been extended to polygonal objects in [4]. A new set of qualitative dynamic characteristic of the motion are derived to relate the initial velocities and the displacement of the object. Heuristic rules were developed to search for the desired initial velocities in the 3-dimension initial velocity space. In above two approaches, first qualitative relationships between the initial velocities and displacement were derived, then the searching algorithm for the velocities are developed through bisection of the initial velocity space. All of above methods depends on some characteristics of the motion, they can only be applied to a specific class of objects, Besides, the algorithms used only qualitative heuristic information, the convergence is always slow.

In stead of using qualitative information, this paper proposed a new method to solve the free sliding problem using optimization techniques. Under Coulomb's assumptions on friction, a set of differential equations that govern the motion of the object can be derived for a given object. In the free sliding problem, the initial configuration and final goal configuration are known, and we know that at final goal configuration, the velocities of the object are zero, while the traveling time of the object is unspecified. Based on this observation, the free sliding problem can be formulated as a free boundary value problem. In order to use the well known existing techniques, the problem is reduced to standard two-point boundary value problem and solved by using simple shooting methods[1]. The shooting method is implemented by integrating the initial value solver with optimization routines. Here we use the Quasi-Newton's method as the optimization routine. In general shooting methods, computing of Jacobian in optimization routine is always time consuming because several initial value problems have to be solved. In order to reduce the cost for computing Jacobian, we implement the Quasi-Newton's method with the Broyden update. In which the Jacobian is updated recursively without solving the initial value problems in each iteration.

The method proposed here is different from the previous methods[6][4]. In previous methods, first qualitative relationships between the initial velocities and displacement were derived based on the equations of free sliding objects, then the searching algorithm for the velocities are developed through bisection of the initial velocity space. These methods utilized same strategy for specific type ob-

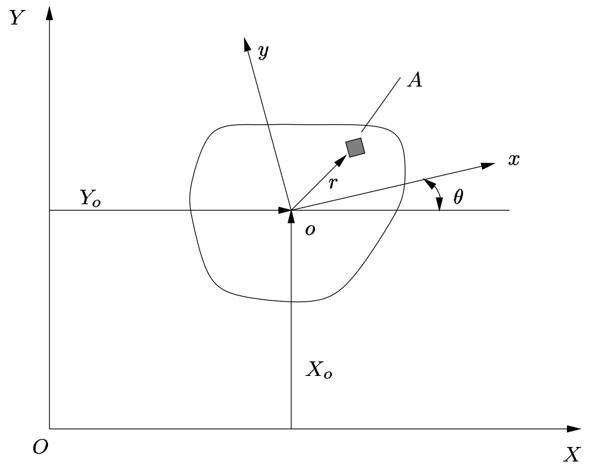

Fig. 1. Motion of planar object

jects or parts which satisfy the qualitative criteria obtained through analysis. For other kind of objects with different geometry or different pressure distribution, new criteria need to be derived to direct the search direction for the initial velocities. Just as the bisection method for non-linear equation, the convergence speed of these methods is always slow. In this paper, instead of using qualitative properties of the motion, quantitative information is used to find desired initial velocities. The proposed method has higher convergence speed and can be applied to more general objects.

The remainder of the paper is organized as follows: model of the sliding motion is derived in section 2, in section 3 we formulate the velocity planning problem as free boundary value problem, and give the standard two-point boundary value(TPBV) formation. Section 4 presents the planning algorithm and implementation. Simulation results are carried out in section 5, and section 6 concludes the paper. And the computation of mass moment of inertia and friction for general objects is discussed in appendix.

II. Model of sliding motion

The motion of an free sliding object on a horizontal plane is governed by the friction force between the object and the plane. The friction force and torque can be calculated by integrating through each infinitesimal element of the object. Consider an object on the plane as shown in Fig. 1. XOY is the global coordinate, xoy is local coordinate associated with the object, and assign o at the center of mass. The configuration space of the object is defined as (X_o, Y_o, θ). Where (X_o, Y_o) gives the position of the local coordinate, while θ denotes the orientation of the object.

Assume the linear velocity of o is $v = (\dot{X}_o, \dot{Y}_o)$, we assume that on each infinitesimal element of the object, there acts a force of friction. Denote A as infinitesimal element of the object located at (x, y) in local coordinate, and r is the position vector from o to A, dm is the mass of element A calculated as $dm = \rho(x,y)dxdy$, $\rho(x,y)$ is the pressure distribution function over the area. $\omega = \dot{\theta}$ the angular velocity of object in the count-clockwise direction, μ is the friction coefficient, g is the acceleration of gravity. By integrating friction forces and torque over overall contact area, the frictional forces and torque acting on the object can be expressed as

$$F_X = -\mu \cdot g \cdot \rho \int\int \frac{A_x}{\sqrt{B}} dxdy \quad (1)$$

$$F_Y = -\mu \cdot g \cdot \rho \int\int \frac{A_y}{\sqrt{B}} dxdy \quad (2)$$

$$T = -\mu \cdot g \cdot \rho \int\int \frac{A_t}{\sqrt{B}} dxdy \quad (3)$$

Where

$$A_x = \dot{X}_o - \cos\theta \cdot \omega \cdot y - \sin\theta \cdot \omega \cdot x \quad (4)$$

$$A_y = \dot{Y}_o - \sin\theta \cdot \omega \cdot y + \cos\theta \cdot \omega \cdot x \quad (5)$$

$$\begin{aligned} A_t &= -(\dot{X}_o x + \dot{Y}_o y)\sin\theta \\ &\quad -(\dot{X}_o y - \dot{Y}_o x)\cos\theta + \omega \cdot y^2 + \omega \cdot x^2 \end{aligned} \quad (6)$$

$$\begin{aligned} B &= (\dot{X}_o - \cos\theta \cdot \omega \cdot y - \sin\theta \cdot \omega \cdot x)^2 \\ &\quad +(\dot{Y}_o - \sin\theta \cdot \omega \cdot y + \cos\theta \cdot \omega \cdot x)^2 \end{aligned} \quad (7)$$

The motion of the sliding object subject to the friction can be written as

$$\begin{aligned} \ddot{X}_o &= F_X/m \\ \ddot{Y}_o &= F_Y/m \\ \ddot{\theta} &= T/I \end{aligned} \quad (8)$$

where m is the mass of the object, while I is the mass moment of inertia refer to the center of mass. Denote $\overline{x} = (x_1, x_2, x_3, x_4, x_5, x_6) = (\dot{X}_o, X_o, \dot{Y}_o, Y_o, \dot{\theta}, \theta)$ as the state variable, (8) can be rewritten as the state space form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{F_X}{m} \\ x_4 \\ \frac{F_Y}{m} \\ x_6 \\ \frac{T}{I} \end{bmatrix} = \begin{bmatrix} f_1(\overline{x}) \\ f_2(\overline{x}) \\ f_3(\overline{x}) \\ f_4(\overline{x}) \\ f_5(\overline{x}) \\ f_6(\overline{x}) \end{bmatrix} \quad (9)$$

If we want to plan to desired initial velocities for free sliding object using the model (8), we must set up a small ϵ, and consider the object stopped when $\sqrt{\dot{X}^2 + \dot{Y}^2 + \dot{\theta}^2} = 0$, this gives us the traveling time T_f.

III. Formulation of the planning problem

The planning problem can be formulated as: An object with known geometry, mass distribution, and friction properties slides on a supporting surface. It starts from initial configuration (X_i, Y_i, θ_i), slowing down to rest at a final configuration (X_f, Y_f, θ_f). Given the initial and final configurations, determine the initial velocities.

The motion of the object on a plane is governed by (9), which is a system of 6 first-order differential equations. There exists no analytical solution for determining the initial velocities for a given displacement. However the initial velocities can only be found by using numerical procedures.

For a given initial configuration (X_i, Y_i, θ_i), and velocities $(\dot{X}_i, \dot{Y}_i, \dot{\theta}_i)$ at time $t = 0$, the trajectory of the motion can be uniquely determined, such that the object occupies the final configuration (X_f, Y_f, θ_f). The solution can be found by numerical integrating the system (9). This is known as the initial value problem. However for the planning problem, the initial configuration (X_i, Y_i, θ_i) at $t = 0$ and final goal configuration (X_f, Y_f, θ_f) are known, while the traveling time T_f of the object is unknown. If both the starting time and ending time for (9) are specified, the problem becomes a standard two-point boundary value (TPBV) problem in differential equation literature[1]. Since in the planning problem, only starting time $t = 0$ is specified, we need to use another boundary condition to determine the ending time T_f, the condition is that the velocities at the ending point T_f decay to zero. Here the system (9) has a solution satisfying 7 boundary conditions

$$\left\{ \begin{array}{ll} X_o(0) = X_i & X_o(T_f) = X_f \\ Y_o(0) = Y_i & Y_o(T_f) = Y_f \\ \theta_o(0) = \theta_i & \theta_o(T_f) = \theta_f \\ \dot{X}_o(T_f) = 0 & \end{array} \right. \tag{10}$$

The statement of this problem is referred as *free boundary value problem.*

The free boundary value problem can be transformed to a TPBV problems by introducing new independent variables. Here in place of time t, we introduce a new independent variable τ, such that

$$t = \tau T_f \quad 0 \le \tau \le 1 \tag{11}$$

$$\dot{T}_f = \frac{dT_f}{d\tau} = 0 \tag{12}$$

In (11), we know that when τ varies from $\tau = 0$ to $\tau = 1$, the system will travel from $t = 0$ to $t = T_f$. And T_f is independent to τ.

After substituting (11) into (9), and augmenting (12) to (9), the motion of the object respect to τ can be written as

$$\left\{ \begin{array}{lcl} \frac{dx_1}{d\tau} & = & T_f \cdot f_1(\overline{x}) \\ \frac{dx_2}{d\tau} & = & T_f \cdot f_2(\overline{x}) \\ & \vdots & \\ \frac{dx_6}{d\tau} & = & T_f \cdot f_6(\overline{x}) \\ \dot{T}_f & = & 0 \end{array} \right. \tag{13}$$

The system of 7 differential equations is now in standard TPBV form with τ varying between the known limits 0 and 1. And the boundary conditions are

$$\left\{ \begin{array}{ll} X_o(0) = X_i & X_o(1) = X_f \\ Y_o(0) = Y_i & Y_o(1) = Y_f \\ \theta_o(0) = \theta_i & \theta_o(1) = \theta_f \\ \dot{X}_o(T_f) = 0 & \end{array} \right. \tag{14}$$

The planning problem is to determine initial velocities $(\dot{X}_o(0), \dot{Y}_o(0), \dot{\theta}(0))$ and the traveling time T_f, such that the solution of (13) satisfy boundary conditions (14).

The methods for solving TPBV problems fall into three categories: (1). the shooting method, (2) the difference method, (3). the variational method[8]. The shooting method is an extension of the initial value techniques. Its advantages are conceptual simplicity and it allows taking advantage of available initial value ordinary differential equations solvers.

IV. Planning the velocities of free sliding

A. Planning algorithm

In this section, the initial velocities are planned using shooting method to the TPBV formulation (13) and (14).

The associated initial value problem is defined as

$$\left\{ \begin{array}{lcl} \frac{dx_1}{d\tau} & = & T_f \cdot f_1(\overline{x}) \\ \frac{dx_2}{d\tau} & = & T_f \cdot f_2(\overline{x}) \\ & \vdots & \\ \frac{dx_6}{d\tau} & = & T_f \cdot f_6(\overline{x}) \\ \frac{dT_f}{d\tau} & = & 0 \end{array} \right. \tag{15}$$

with the initial conditions at $\tau = 0$,

$$\left\{ \begin{array}{ll} X_o(0) = X_i & \dot{X}_o(0) = \dot{X}_i \\ Y_o(0) = Y_i & \dot{Y}_o(0) = \dot{Y}_i \\ \theta_o(0) = \theta_i & \dot{\theta}_o(0) = \dot{\theta}_i \\ T_f(0) = T_{fi} & \end{array} \right. \tag{16}$$

We know from the boundary condition (14) that $X_o(0) = X_i, Y_o(0) = Y_i, \theta_o(0) = \theta_i$ are known, and the configuration $X_o(1), Y_o(1), \theta_o(1)$ and the linear velocity $\dot{X}_o(1)$ at $\tau = 1$ are functions of $s = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)^T = (\dot{X}_o(0), \dot{Y}_o(0), \dot{\theta}(0), T_f(0))$ at $\tau = 0$ in initial value problem. In order to solve the TPBV problem of (13), (14), we need to determine a starting velocities and traveling time $T_f(0)$ as $s = (\dot{X}_o(0), \dot{Y}_o(0), \dot{\theta}(0), T_f(0))$ for the initial value problem (15), (16), such that the solution obeys the boundary conditions (14) at the other end $\tau = 1$ as

$$\begin{array}{ll} X_o(1, s) = X_f & Y_o(1, s) = Y_f \\ \theta_o(1, s) = \theta_f & \dot{X}_o(1, s) = 0 \end{array} \tag{17}$$

rewriting (17) as a vector function form

$$F(s) = \begin{array}{c} [\ X_o(1, s) - X_f \quad Y_o(1, s) - Y_f \\ \theta_o(1, s) - \theta_f \quad \dot{X}_o(1, s)\ ]^T \end{array} \tag{18}$$

where $s = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)^T$.

Solving the TPBV problem is equivalent to finding a solution of s as $\overline{s} = (\overline{\sigma}_1, \overline{\sigma}_2, \overline{\sigma}_3, \overline{\sigma}_4)$ such that

$$F(\overline{s}) = 0 \tag{19}$$

This nonlinear equations (18) can be solved by means of the general Newton's method

$$s^{(i+1)} = s^{(i)} - DF(s^{(i)})^{-1} \cdot F(s^{(i)}) \tag{20}$$

In each iteration step, one has to compute $F(s^{(i)})$, and the Jacobian matrix

$$DF(s^{(i)}) = \left[\frac{\partial F_j}{\partial \sigma_k} \right]_{s = s^{(i)}} \tag{21}$$

and the solution $d^{(i)} = s^{(i)} - s^{(i+1)}$ of the linear system of equation $DF(s^{(i)})d^{(i)} = F(s^{(i)})$. For the computation of $F(s^{(i)})$, one must solve the initial value problem (15),(16) for $s = s^{(i)} = (\sigma_1^{(i)}, \sigma_2^{(i)}, \sigma_3^{(i)}, \sigma_4^{(i)})$, the Jacobian $DF(s^{(i)})$ can not be calculated analytically, and it will be approximated by the matrix

$$\Delta F(s^{(i)}) = \left[\begin{array}{ccc} \Delta F_1(s^{(i)}) & \cdots & \Delta F_2(s^{(i)}) \end{array}\right] \tag{22}$$

where

$$\begin{array}{rl} \Delta F_j(s^{(i)}) & = \frac{1}{\Delta\sigma_j^{(i)}}(F_j(\sigma_1^{(i)}, \cdots, \sigma_j^{(i)} + \Delta\sigma_j^{(i)}, \cdots, \sigma_4^{(i)}) \\ & -F_j(\sigma_1^{(i)}, \cdots, \sigma_j^{(i)}, \cdots, \sigma_4^{(i)})) \\ & \text{for} \quad j = 1, 2, 3, 4 \end{array} \tag{23}$$

As the computation of $F(s^{(i)})$, the calculation of $F(\sigma_1^{(i)}, \cdots, \sigma_j^{(i)} + \Delta\sigma_j^{(i)}, \cdots, \sigma_4^{(i)})$ requires to solve the corresponding initial value problems (15), (16) with initial conditions $s = (\sigma_1^{(i)}, \cdots, \sigma_j^{(i)} + \Delta\sigma_j^{(i)}, \cdots, \sigma_4^{(i)})$. The approximate Newton's method is carried out as

$$s^{(i+1)} = s^{(i)} - \Delta F(s^{(i)})^{-1} \cdot F(s^{(i)}) \tag{24}$$

The planning algorithm is summarized as
1. Choose a starting vector $s^{(0)}$,
For $i = 0, 1, 2, \cdots$, repeat steps 2-4,
2. Determine $X_o(1, s^{(i)}), Y_o(1, s^{(i)}), \theta_o(1, s^{(i)}), \dot{X}_o(1, s^{(i)})$ by solving initial value problem (15), (16), then compute $F(s^{(i)})$ according to (18).
3. Choose $\Delta\sigma_j, j = 1, \cdots, 4$, and determine $X_o(1, s^{(i)} + \Delta\sigma_j e_j), Y_o(1, s^{(i)} + \Delta\sigma_j e_j), \theta_o(1, s^{(i)} + \Delta\sigma_j e_j), \dot{X}_o(1, s^{(i)} + \Delta\sigma_j e_j)$ by solving 4 initial value problems (15), (16) for

$$s = s^{(i)} + \Delta\sigma_j e_j = \left[\sigma_1^{(i)}, \cdots, \sigma_j^{(i)} + \Delta\sigma_j, \cdots, \sigma_4^{(i)}\right]^T \tag{25}$$

where e_j is a 4 dimensional vector of following form

$$e_j(i) = \left\{ \begin{array}{ll} 1 & i = j \\ 0 & i \neq j \end{array} \right. \tag{26}$$

4. Compute $\Delta F(s^{(i)})$ by means of (22),(23), and also the solution $d^{(i)}$ of the system of linear equations

$$\Delta F(s^{(i)})d^{(i)} = -F(s^{(i)}) \tag{27}$$

and update

$$s^{(i+1)} = s^{(i)} + d^{(i)} \tag{28}$$

Algorithm 1: Shooting method without Broyden's update

In each step of the method, 5 initial value problems and a 4-th order system of linear equation need to be solved.

B. Shooting with Broyden's Update

In each iteration of the algorithm, the Jacobian is computed numerically according to (22),(23), which needs solving 4 initial value problem. The computing of initial value problem is always time consuming. In order to reduce the computational cost, we consider to use the Broyden's method[2] to approximate the Jacobian matrix, the local convergence has been proven in [2]. In Broyden's method, The Jacobian matrix is updated in each iteration instead of computing numerically using (22) and (23). The planning algorithm with Broyden's update is:
1. Choose a starting vector $s^{(0)}$,
2. Determine $X_o(1, s^{(0)}), Y_o(1, s^{(0)}), \theta_o(1, s^{(0)}), \dot{X}_o(1, s^{(0)})$ by solving initial value problem, then compute $F(s^{(0)})$ according to (18).
3. Choose $\Delta\sigma_j, j = 1, \cdots, 4$, and determine $X_o(1, s^{(0)} + \Delta\sigma_j e_j), Y_o(1, s^{(0)} + \Delta\sigma_j e_j), \theta_o(1, s^{(0)} + \Delta\sigma_j e_j), \dot{X}_o(1, s^{(0)} + \Delta\sigma_j e_j)$ by solving 4 initial value problems for

$$s_j^{(0)} = s^{(0)} + \Delta\sigma_j e_j = \left[\sigma_1^{(0)}, \cdots, \sigma_j^{(0)} + \Delta\sigma_j, \cdots, \sigma_4^{(0)}\right]^T \tag{29}$$

where e_j is a 4 dimensional vector of following form

$$e_j(i) = \left\{ \begin{array}{ll} 1 & i = j \\ 0 & i \neq j \end{array} \right. \tag{30}$$

4. Compute $\Delta F(s^{(0)})$ by means of (22), (23).
For $i = 0, 1, 2, \cdots$, repeat steps 5-8,
5. Compute the solution $d^{(i)}$ of the system of linear equations

$$\Delta F(s^{(i)})d^{(i)} = -F(s^{(i)}) \tag{31}$$

and update

$$s^{(i+1)} = s^{(i)} + d^{(i)} \tag{32}$$

6. Determine $X_o(1, s^{(i+1)}), Y_o(1, s^{(i+1)}), \theta_o(1, s^{(i+1)})$, $\dot{X}_o(1, s^{(i+1)})$ by solving initial value problem (15), (16), then compute $F(s^{(i+1)})$ according to (18).
7. Compute

$$y^{(i)} = F(s^{(i+1)}) - F(s^{(i)}) \tag{33}$$

where $y^{(i)}$ is the difference of the function $F(.)$ during iteration $i+1$ and i, which provide the information for computing Jacobian.
8. Update Jacobian

$$\Delta F(s^{(i+1)}) = \Delta F(s^{(i)}) + \frac{(y^{(i)} - \Delta F(s^{(i)})d^{(i)}(d^{(i)})^T}{(d^{(i)})^T d^{(i)}} \tag{34}$$

Algorithm 2: Shooting method with Broyden's update

In algorithm 1, the computation of Jacobian is approximated by (22), which needs to solve 4 additional initial value problem in each iteration. Instead, the Jacobian is updated through (34) in algorithm 2. This algorithm only needs to solve one initial value problem in each iteration, which reduces the computational cost dramatically.

V. Simulation Results

Simulations are carried out with planning algorithm 2 on polygonal objects with 3-5 vertices, even and uneven pressure distribution are considered.

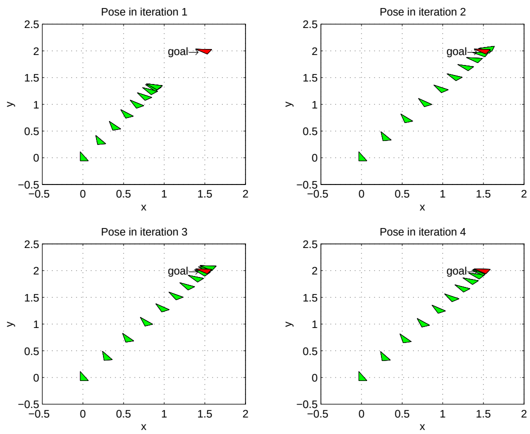

Fig. 2. Trajectory of object vs. iteration

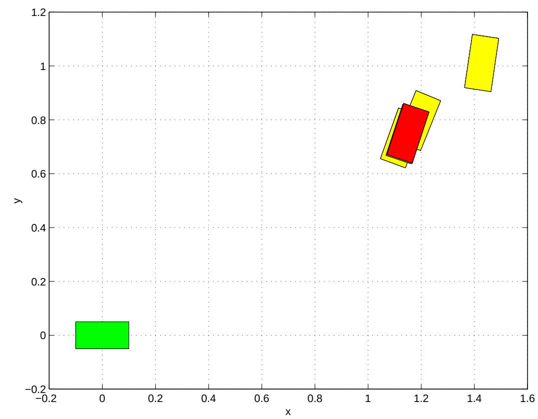

Fig. 3. Final configuration vs. Iteration

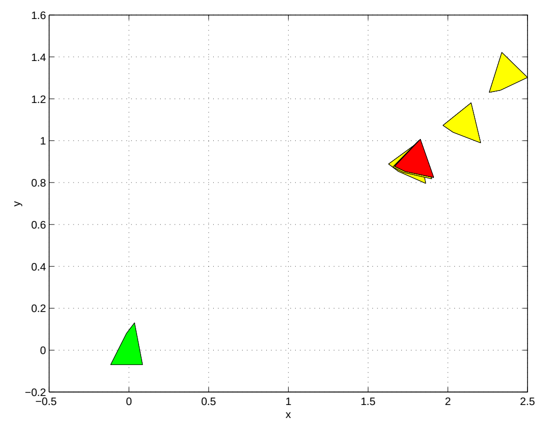

Fig. 4. Final configuration vs. Iteration

A. Planning for object with even pressure distribution

Consider a triangle object with edges $c = 0.2m, b = 0.17m$, angle $A = \pi/6$, pressure distribution $\rho = 200kg/m^2$, friction coefficient $\mu = 0.5$. The mass moment of inertia are calculated using the procedure proposed in appendix A.1, and the integration of the friction on the triangular contact area is carried out according to appendix A1. The initial configuration is $[X_0 \quad Y_0 \quad \theta_0] = [0 \quad 0 \quad 0]$, the goal configuration is $[X_f \quad Y_f \quad \theta_f] = [1.5m \quad 2m \quad 1rad]$. Using the planning algorithm 2 proposed in this paper, the desired velocities were found in 4 iterations, the desired velocities are $[\dot{X} \quad \dot{Y} \quad \omega] = [2.9686m/s \quad 3.9595m/s \quad 1.7257m/s]$, and the trajectories of the object in each iteration are shown in Fig. 2.

Consider a rectangular object with even pressure distribution $\rho(x,y) = 200kg/m^2$, and dimension is $0.2m \times 0.1m$, The goal configuration is $[X_f \quad Y_f \quad \theta_f] = [1.5m \quad 2m \quad 1rad]$. Using the planning algorithm proposed in this paper, with initial guess of velocities as $s^{(0)} = [3.2m/s \quad 2.3m/s \quad 2.8rad/s]$. The configuration vs. iteration is shown in Fig. 3. It is clear that the planner find the desired velocities that lead the object to the goal configuration. The planning algorithm can obtain the desired initial velocities $[\dot{X} \quad \dot{Y} \quad \omega] = [2.88m/s \quad 1.9m/s \quad 2.77m/s]$ in 4 iterations.

Consider a pentagon with vertices located at $(0,0), (0.2,0), (0.175,0.1), (0.15,0.2), (0.1,0.15)$in global coordinate, and the pressure distribution function $rho = 200kg/m^2$, The goal configuration is $[X_f \quad Y_f \quad \theta_f] = [1.8m \quad 0.9m \quad 2.0rad]$. Using the planning algorithm proposed in this paper, with initial guess of velocities as $s^{(0)} = [4.5m/s \quad 2.5m/s \quad 3.5rad/s]$. The configuration vs. iteration is shown in Fig. 4. The planning algorithm can find the desired initial velocities $[\dot{X} \quad \dot{Y} \quad \omega] = [3.97m/s \quad 1.98m/s \quad 3.18m/s]$.

B. Planning for object with uneven pressure distribution

Consider the rectangular object used in last section, and dimension is $0.2m \times 0.1m$, now consider the uneven pressure distribution $rho(x,y) = 100kg/m^2$ for $x < 0.1$ and $rho(x,y) = 200kg/m^2$ for $x > 0.1$. The goal configuration is $[X_f \quad Y_f \quad \theta_f] = [1.8m \quad 1m \quad 2rad]$. Using the planning algorithm proposed in this paper, with initial guess of velocities as $s^{(0)} = [4.5m/s \quad 2.5m/s \quad 3.5rad/s]$. The final configuration vs. iteration is shown in Fig. 5. The planning algorithm can obtain the desired initial velocities $[\dot{X} \quad \dot{Y} \quad \omega] = [3.93m/s \quad 2.19m/s \quad 3.46m/s]$. It is clear that the planning algorithm works also for the objects with uneven pressure distribution.

VI. Conclusion

In this paper, a novel computational approach has been proposed to solve the initial velocities for the free sliding objects on a plane. The velocity planning problem is formulated as a free boundary value problem, and solved by using nonlinear optimization techniques. In the proposed method, quantitative information is used to search the desired velocities, no motion characteristics such as monotonicity of force and torque are needed, this method can be used for objects with general shapes and pressure distribution. The planning method is verified using numerical simulation on polygonal objects with 3-5 vertices under even and uneven pressure distribution. The proposed method gives really quick convergence. The proposed planning

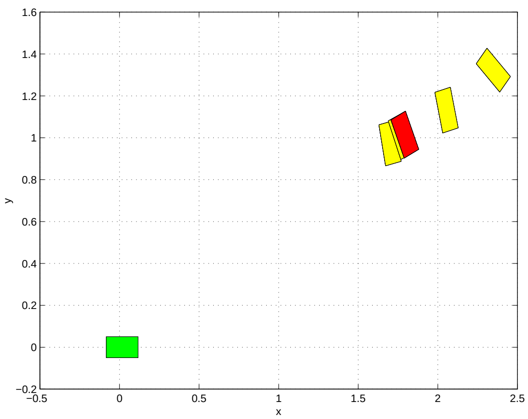

Fig. 5. Final configuration vs. Iteration

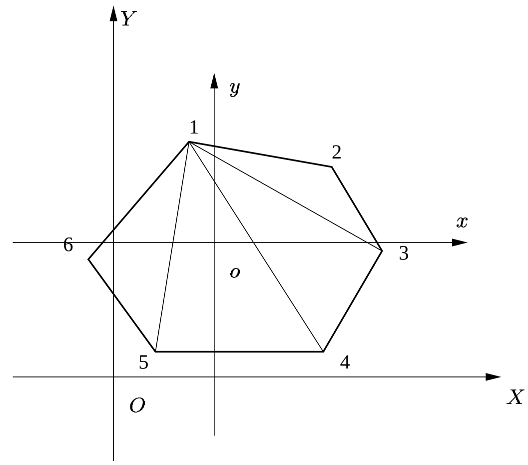

Fig. 6. Geometry of polygonal object

method can be used in any impulse based manipulation or other manipulations which need to solve initial velocities. Experiments need to be done to verify the planning method.

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Appendix

I. Computation of mass moment of inertia and friction

For general geometrical objects, there is no analytical formulation to compute the mass, mass moment of inertia, friction forces and torque. One possible way is to partition the object into a number of simple areas such as triangles, and working out the mass moment of inertia, friction forces and torque of each simple area first, then summing up of them. In next two sections, the computation methods for convex polygonal objects are discussed.

A. Mass moment of inertia of polygon

Consider convex polygon as shown in Fig. 6 in global frame XOY. Mark the vertices as $V_1, V_2, V_3, \cdots, V_n$, with the coordinate (X_i, Y_i) of V_i in global frame, the mass pressure distribution function is $\rho_1(X,Y)$. Convex polygons can be divided into $n-2$ triangles as

$$V_1V_2V_3, V_1V_3V_4, V_1V_4V_5, \cdots, V_1V_kV_{k+1}, \cdots, V_1V_{n-1}V_n$$

. and labeled as $T_1, T_2, \cdot, T_{n-2}$. In order to planning the velocities for free sliding object, we need to know the mass m, mass moment of inertia I of the object, and also the center of mass. All these quantities can be computed numerically by integrating over the triangles $T_1, T_2, \cdots, T_{n-2}$.

The mass of the object can be calculated as

$$\begin{aligned} m &= \int\int_S \rho_1(X,Y)dXdY \\ &= \sum_{i=1}^{n-2} \int\int_{T_i} \rho_1(X,Y)dXdY \end{aligned} \tag{35}$$

The center of mass (X_o, Y_o) can be computed as

$$X_o = m_x/m \qquad Y_o = m_y/m \tag{36}$$

Where m_x and m_y are computed as

$$\begin{aligned} m_y &= \int\int_S X\rho_1(X,Y)dXdY \\ &= \sum_{i=1}^{n-2} \int\int_{T_i} X\rho_1(X,Y)dXdY \end{aligned} \tag{37}$$

$$\begin{aligned} m_y &= \int\int_S Y\rho_1(X,Y)dXdY \\ &= \sum_{i=1}^{n-2} \int\int_{T_i} Y\rho_1(X,Y)dXdY \end{aligned} \tag{38}$$

After the center of mass (X_o, Y_o) are computed, assign a local coordinate passing through the center of mass as xoy. The coordinates of vertices of the polygon can be represented in local coordinate as

$$x_i = X_i - X_o \quad y_i = Y_i - Y_o \tag{39}$$

And the mass pressure distribution function becomes $\rho(x,y) = \rho_1(x + X_o, y + Y_o)$ in xoy coordinate. The mass moment of inertia I with respect to the center of mass can be computed as

$$I = I_x + I_y \tag{40}$$

where

$$\begin{aligned} I_x &= \int\int_S y^2\rho(x,y)dxdy \\ &= \sum_{i=1}^{n-2} \int\int_{T_i} y^2\rho(x,y)dxdy \end{aligned} \tag{41}$$

$$\begin{aligned} I_y &= \int\int_S x^2\rho(x,y)dxdy \\ &= \sum_{i=1}^{n-2}\int\int_{T_i} x^2\rho(x,y)dxdy \end{aligned} \quad (42)$$

In order to compute the mass, center of mass, mass moment of inertia of the polygonal object, double integrations (35), (37), (38), (41), (42) need to be carried out over the triangular regions T_i. A numerical procedure is developed in next section.

B. Numerical double integration over triangular region

In order to implement shooting method to the planning problem, we also need to compute the friction forces and torque described by (1), (2), (3) in the contact area. The computation of friction force and torque needs the double integration of (1), (2), (3) on the contact area. For general geometrical object the double integration can only solved by numerical methods.

Consider a triangle $T_i, i = 1, \cdots, n-2$, assign the vertices as $(x_1, y_1), (x_2, y_2), (x_3, y_3)$. Almost all existing double integration routines work only under rectangular boundaries, and they can not be used to solve the integration for the irregular contact boundaries, we need to find transformations to transfer the irregular boundaries into rectangular boundaries such that we can utilize the existing routines. Here consider following transformation

$$\begin{aligned} x &= x_1(-\tfrac{1}{2}(\xi+\eta)) + x_2(\tfrac{1}{2}(1+\xi)) + x_3(\tfrac{1}{2}(1+\eta)) \\ y &= y_1(-\tfrac{1}{2}(\xi+\eta)) + y_2(\tfrac{1}{2}(1+\xi)) + y_3(\tfrac{1}{2}(1+\eta)) \end{aligned} \quad (43)$$

maps a triangle with vertices (x_i, y_i) into the standard triangle with vertices $(-1,-1), (1,-1), (-1,1)$ in the $\xi\eta$ plane. The Jacobian associated with the transformation is

$$\begin{aligned} J &= \begin{vmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{vmatrix} \\ &= \begin{vmatrix} \frac{1}{2}(-x_1+x_2) & \frac{1}{2}(-y_1+y_2) \\ \frac{1}{2}(-x_1+x_3) & \frac{1}{2}(-y_1+y_3) \end{vmatrix} \end{aligned} \quad (44)$$

By substituting (43) and (44) into (1), (2), (3), the integrand become $fx(\xi,\eta), fy(\xi,\eta), t(\xi,\eta)$, and we get following integration formula

$$\begin{aligned} F_X &= -\mu \cdot g \int_{-1}^{1}\int_{-1}^{-x} J\cdot\rho(x,y)\cdot fx(\xi,\eta)d\eta d\xi \\ F_Y &= -\mu \cdot g \int_{-1}^{1}\int_{-1}^{-x} J\cdot\rho(x,y)\cdot fy(\xi,\eta)d\eta d\xi \\ T &= -\mu \cdot g \int_{-1}^{1}\int_{-1}^{-x} J\rho(x,y)\cdot\cdot t(\xi,\eta)d\eta d\xi \end{aligned} \quad (45)$$

Till now, we transform the integration on a arbitrary triangle area into a standard triangle area. In order to transfer the triangular area into a rectangular area, here we introduce another transformation

$$\eta = (-\xi+1)z - 1 \quad (46)$$

Thus when $z = 1, \eta = -\xi$ and when $z = 0, \eta = -1$ as required in (45). Differentiating the above expression we have

$$d\eta = (-\xi+1)dz$$

substituting (46) and $d\eta$ into (45), we have

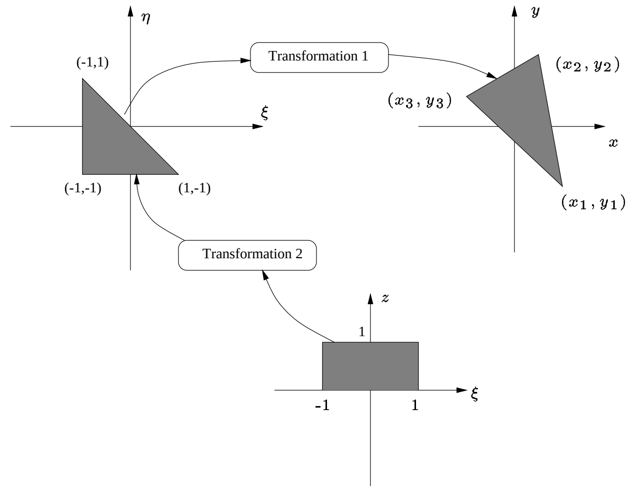

Fig. 7. Geometry of triangular object

$$\begin{aligned} F_X &= -\mu \cdot g \int_{-1}^{1}\int_{0}^{1} J\rho(x,y)\cdot fx1(\xi,z)dzd\xi \\ F_Y &= -\mu \cdot g \int_{-1}^{1}\int_{0}^{1} J\rho(x,y)\cdot fy1(\xi,z)dzd\xi \\ T &= -\mu \cdot g \int_{-1}^{1}\int_{0}^{1} J\rho(x,y)\cdot t1(\xi,z)dzd\xi \end{aligned} \quad (47)$$

These integrals now become standard form with rectangular boundaries, and they can be integrated using existing double integral routines such as Gaussian quadrature directly. The transformations are shown in Fig. 7.

For general objects, after we get all the frictions and torque on each triangular areas, the overall friction force and torque can be obtained easily by summation.

The computation of double integrations (35), (37), (38), (41), (42) can be carried out in same manner as above.